Polaritons

$$|\psi(t)\rangle = C_g(t)\,|g\rangle + C_e(t)\,|e\rangle$$

$$P_g(t) = |C_g(t)|^2 \qquad P_e(t) = |C_e(t)|^2$$

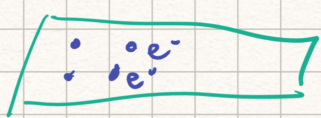
$$P_g(t) + P_e(t) = 1 = |C_g(t)|^2 + |C_e(t)|^2$$

Polariton system

Strong coupling between a photon and an exciton

Exciton =

hole: absence on an electron



Photons and excitons are bosons

↳ described as Q. harmonic oscilators

a: annihilation operator of the photon field

b: annihilation operator of the exciton field

$$[a, a^\dagger] = 1 \qquad [x, p] = i\hbar$$

$$[b, b^\dagger] = 1$$

$$[a, b] = [a, b^\dagger] = [a^\dagger, b] = [a^\dagger, b^\dagger] = 0$$

$$|\psi\rangle = \sum_{n_p}{}' \sum_{n_e}{}' C_{n_p, n_e} \underbrace{|n_p\rangle}_{\text{photons}} \underbrace{|n_e\rangle}_{\text{excitons}}$$

n_p photons
n_e excitons

$$= \sum_{n_p}{}' \sum_{n_e}{}' C_{n_p, n_e} |n_p, n_e\rangle$$

(n_p → photon, n_e → exciton)

$|n_p, n_e\rangle$: Q. state of a photon and an exciton field.

Example:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\underset{\substack{1\text{ photon}\\ 0\text{ excitons}}}{|1,0\rangle} + \underset{\substack{0\text{ photons}\\ 1\text{ exciton}}}{|0,1\rangle}\right)$$

$$H_0 = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$a|k\rangle = \sqrt{k}\,|k-1\rangle$$

$$\langle k|a^\dagger = \langle k-1|\sqrt{k}$$

$$\langle 0|H_0|0\rangle = \hbar\omega\,\langle 0|\left(a^\dagger a + \frac{1}{2}\right)|0\rangle$$

$$= \hbar\omega\left(\cancelto{0}{\langle 0|a^\dagger a|0\rangle} + \frac{1}{2}\underbrace{\langle 0|0\rangle}_{1}\right)$$

$$= \frac{1}{2}\hbar\omega$$

We are not interested in absolute energies but on energy differences

$$E_1 = \frac{1}{2}\hbar\omega_0 + \hbar\omega$$

$$\Delta E = E_1 - E_0 = \hbar\omega$$

$$E_0 = \frac{1}{2}\hbar\omega_0$$

Energy of a free boson $\Rightarrow$ $H_0 = \hbar\omega\, a^\dagger a$

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + g\left(a^\dagger b + b^\dagger a\right)$$

$a^\dagger b$: remove an exciton and add a photon
an exciton becomes a photon

$b^\dagger a$: remove a photon and add an exciton
a photon becomes an exciton

- Time-dependent S.E.

$$i\hbar\, \partial_t |\psi(t)\rangle = H |\psi(t)\rangle$$

$$H = \hbar\omega_a \underbrace{a^\dagger a}_{\substack{\text{\# photons}\\ \text{remains}\\ \text{constant}}} + \hbar\omega_b \underbrace{b^\dagger b}_{\substack{\text{\# excitons}\\ \text{remains}\\ \text{constant}}} + \hbar g\,(a^\dagger b + b^\dagger a)$$

5 photons 5 exciton
10 particles

6 photons 4 excitons

4 photons 6 excitons

H keeps the number of particles constant.

Example:

$|\psi(t=0)\rangle = |1,0\rangle$ $\qquad \{ |1,0\rangle, |0,1\rangle \}$

$$|\psi(t)\rangle = C_p(t)\,|1,0\rangle + C_e(t)\,|0,1\rangle$$

$a|k\rangle = \sqrt{k}|k-1\rangle$

$a^\dagger|k-1\rangle = \sqrt{k}|k\rangle$

$a^\dagger a|k\rangle = k|k\rangle$

$$a^\dagger a\,|\psi(t)\rangle = C_p(t)\,|1,0\rangle$$
$$b^\dagger b\,|\psi(t)\rangle = C_e(t)\,|0,1\rangle$$
$$a^\dagger b\,|\psi(t)\rangle = C_e(t)\,|1,0\rangle$$
$$b^\dagger a\,|\psi(t)\rangle = C_p(t)\,|0,1\rangle$$

$$H|\psi(t)\rangle = \hbar\omega_a\, a^\dagger a\,|\psi(t)\rangle + \hbar\omega_b\, b^\dagger b\,|\psi(t)\rangle + \hbar g\,(a^\dagger b + b^\dagger a)|\psi(t)\rangle$$

$$= \hbar\omega_a\, C_p(t)\,|1,0\rangle + \hbar\omega_b\, C_e(t)\,|0,1\rangle + \hbar g\,\big(C_e(t)\,|1,0\rangle + C_p(t)\,|0,1\rangle\big)$$

$$= \hbar\big(\omega_a C_p(t) + g\,C_e(t)\big)\,|1,0\rangle + \hbar\big(\omega_b C_e(t) + g\,C_p(t)\big)\,|0,1\rangle$$

$$i\hbar\,\partial_t |\psi(t)\rangle = i\hbar \frac{\partial}{\partial t}\left\{ C_p(t)\,|1,0\rangle + C_e(t)\,|0,1\rangle \right\}$$

$$= i\hbar \frac{\partial C_p}{\partial t}\,|1,0\rangle + i\hbar \frac{\partial C_e}{\partial t}\,|0,1\rangle$$

$$i\hbar \frac{\partial C_p}{\partial t} = \hbar\left(w_a C_p(t) + g\, C_e(t)\right)$$

$$i\hbar \frac{\partial C_e}{\partial t} = \hbar\left(w_b C_e(t) + g\, C_p(t)\right)$$

Divide on both sides by $\frac{1}{i\hbar}$

$\frac{1}{i} = -i$

$$\boxed{\begin{aligned}\frac{\partial C_p}{\partial t} &= -i\left(w_a C_p(t) + g\, C_e(t)\right)\\ \frac{\partial C_e}{\partial t} &= -i\left(w_b C_e(t) + g\, C_p(t)\right)\end{aligned}}$$

$$\begin{aligned}\partial_t C_p &= -i w_a C_p - i g C_e\\ \partial_t C_e &= -i g C_p - i w_b C_e\end{aligned} \Rightarrow \partial_t \begin{pmatrix} C_p \\ C_e \end{pmatrix} = -i \begin{pmatrix} w_a & g \\ g & w_b \end{pmatrix} \begin{pmatrix} C_p \\ C_e \end{pmatrix}$$

$$\partial_t \vec{v} = M \vec{v} \Rightarrow \vec{v}(t) = e^{Mt}\,\vec{v}(0)$$

$$e^{Mt} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

$$(w_a - w_b) = \Delta \qquad R = \sqrt{4g^2 + \Delta^2}$$

$$m_{11} = \frac{1}{2R}\left[R + \Delta + e^{iRt}(R - \Delta)\right] e^{-it(R + w_a + w_b)/2}$$

$$m_{12} = -\frac{2i}{R}\, g \sin\left(\frac{Rt}{2}\right) e^{-it(w_a + w_b)/2}$$

$$m_{21} = -\frac{2i}{R}\, g \sin\left(\frac{Rt}{2}\right) e^{-it(w_a + w_b)/2}$$

$$m_{22} = \frac{1}{2R}\left[R-\Delta + (R+\Delta)e^{iRt}\right] e^{-it(R+\omega_a+\omega_b)/2}$$

$$\begin{pmatrix} C_p(t) \\ C_e(t) \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} C_p(0) \\ C_e(0) \end{pmatrix}$$

$$|\psi(t)\rangle = C_p(t)\,|1,0\rangle + C_e(t)\,|0,1\rangle$$

In our example $C_p(0) = 1$ $C_e(0) = 0$

$$C_p(t) = m_{11} = \frac{1}{2R}\left[R+\Delta + e^{iRt}(R-\Delta)\right] e^{-it(R+\omega_a+\omega_b)/2}$$

$$C_e(t) = m_{21} = -\frac{2i}{R}\, g \sin\left(\frac{Rt}{2}\right) e^{-it(\omega_a+\omega_b)/2}$$

$$P_p(t) = |C_p(t)|^2 = \frac{1}{2R}\left[R+\Delta + e^{iRt}(R-\Delta)\right]\frac{1}{2R}\left[R+\Delta + e^{-iRt}(R-\Delta)\right]$$

$$= \frac{1}{4R^2}\left[(R+\Delta)^2 + (R-\Delta)^2 + (R-\Delta)(R+\Delta)\left(e^{iRt} + e^{-iRt}\right)\right]$$

$$= \frac{1}{4R^2}\left\{2(R^2+\Delta^2) + 2(R^2-\Delta^2)\cos(Rt)\right\}$$

$$= \frac{1}{2R^2}\left\{R^2+\Delta^2 + (R^2-\Delta^2)\cos(Rt)\right\}$$

$$R = \sqrt{4g^2+\Delta^2}$$

$$R^2 - \Delta^2 = 4g^2 + \Delta^2 - \Delta^2$$
$$= 4g^2$$
$$R^2 + \Delta^2 = 4g^2 + \Delta^2 + \Delta^2$$
$$= 4g^2 + 2\Delta^2 = 2(2g^2+\Delta^2)$$

$$P_p(t) = \frac{1}{2R^2}\left\{2(2g^2+\Delta^2) + 4g^2\cos(Rt)\right\}$$

$$= \frac{1}{R^2}\left\{ 2g^2 + \Delta^2 + 2g^2\cos(Rt) \right\}$$

$g = 1$
Plot as function of t for several Δ

We have found Rabi oscillations in the polaritons

Frequency of oscillation : Rabi oscillation

$$R = \sqrt{4g^2 + \Delta^2}$$