Schrödinger's Equation

Wave function : $\psi(\vec{r}, t)$ $\quad$ $\vec{r} = (x, y, z)$: Space

$\downarrow$ $\quad$ t: time

Psi

- $\psi(\vec{r}, t)$ is a complex-valued function

In 1D:

$\psi(x, t)$: Wavefunction of a quantum object in 1D

Associated to the position of quantum particle (e.g., an electron) in a wire.

Interpretation:

The modulus squared of $\psi(\vec{r}, t)$ is the probability distribution

$$|\psi(\vec{r}, t)|^2 = \psi(\vec{r}, t)\, \psi^*(\vec{r}, t)$$

complex conjugate of $\psi(\vec{r}, t)$

$|\psi(\vec{r}, t)|^2 \delta x \delta y \delta z$ is the probability to find the electron between

x and $x + \delta x$; y and $y + \delta y$; z and $z + \delta z$

$$P(a \leq x \leq b) = \int_a^b |\psi(x, t)|^2 dx$$

Prob. to find the particle between a and b.

Normalization condition:

$$\boxed{\int_{-\infty}^{\infty} |\psi(x, t)|^2 dx = 1}$$

We could obtain the average position

$$\langle x \rangle = \int_{-\infty}^{+\infty} x \, |\psi(x,t)|^2 \, dx \qquad f(x) = |\psi(x,t)|^2$$

Where is the particle?

Everything that can be know (measured) about the particle, can be evaluated through $\psi(x,t)$

Physical observables:

- Quantity that can be measured or observed
  - Position
  - velocity → momentum
  - Energy
  - Temperature
  - Angular momentum
  - Polarization

In Q. theory observables → Operators

An operator is a mathematical object that acts onto functions and creates other functions

- Derivative operator

$$D_x = \frac{d}{dx} \qquad D_x f(x) = f'(x)$$

$$f'(x) = \frac{d}{dx} f(x)$$

These operators behave like random variables in probability theory.

- Applying the position op. onto $\psi(x,t) \Rightarrow x\psi(x,t)$

Averages

$$\langle \Omega \rangle = \int \psi^*(x,t)\, \Omega\, \psi(x,t)\, dx$$

A postulate of quantum mechanics

$$\psi^* \Omega \psi \neq \Omega \psi^* \psi$$

: operators and $\psi(x,t)$ do NOT commute in general

For $\Omega = x$

$$\langle x \rangle = \int \psi^*(x,t)\, x\, \psi(x,t)\, dx$$

$$= \int x\, \psi^*(x,t)\psi(x,t)\, dx$$

$$= \int x\, |\psi(x,t)|^2\, dx$$

Because $\psi(x,t)$ is a function of x, $\psi(x,t)$ and x commute

The Schrödinger's Equation

Erwin Schrödinger in 1925

$$\boxed{i\hbar\, \partial_t \psi(x,t) = -\frac{\hbar^2}{2m}\, \partial_x^2 \psi(x,t) + V(x,t)\, \psi(x,t)}$$

$i = \sqrt{-1}$: complex unity

$\hbar$: reduced Planck constant $= \frac{h}{2\pi}$ [J·s]

$\partial_t = \frac{\partial}{\partial t}$

m: mass of the quantum object

$\partial_x^2 = \frac{\partial^2}{\partial x^2}$

$V(x,t)$: potential to which the q. particle is subjected