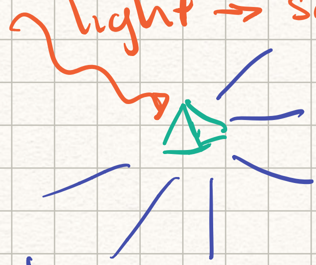
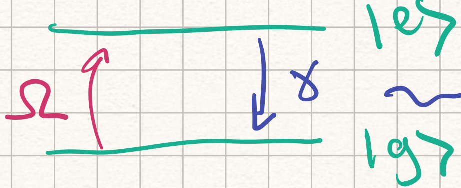
Two-level system

atom

light → source/excitation



luminescence



$\rightsquigarrow \hbar\omega$: photon

Ω: excitation

γ: decay

Oscillation $|g\rangle \Leftrightarrow |e\rangle$

$$|\psi\rangle = c_g |g\rangle + c_e |e\rangle$$

$c_g, c_e \in \mathbb{C}$.

$$P_e = |c_e|^2 \qquad P_g = |c_g|^2$$

$\mathcal{H} = \{|g\rangle, |e\rangle\}$: orthonormal

$$c_e = \langle e|\psi\rangle = \langle e|\left(c_g |g\rangle + c_e |e\rangle\right)$$
$$= c_g \langle e|g\rangle + c_e \langle e|e\rangle$$
$$= c_g \, \emptyset + c_e \cdot 1$$
$$= c_e$$

$$c_e^* = \left(\langle e|\psi\rangle\right)^* = \langle\psi|e\rangle$$

$$c_g = \langle g|\psi\rangle$$

$$c_g^* = \langle\psi|g\rangle$$

$$\langle\psi| = c_g^* \langle g| + c_e^* \langle e|$$

$$c_e^* c_e = \langle\psi|e\rangle\langle e|\psi\rangle \qquad c_g^* c_g = \langle\psi|g\rangle\langle g|\psi\rangle$$

Prob. excited st. prob. ground st.

$$c_e^* c_e + c_g^* c_g = 1 = \langle\psi|e\rangle\langle e|\psi\rangle + \langle\psi|g\rangle\langle g|\psi\rangle$$

$$= \langle\psi|\Big(|e\rangle\langle e| + |g\rangle\langle g|\Big)|\psi\rangle$$

$$\langle\psi|\psi\rangle = 1 = c_e^* c_e + c_g^* c_g$$

$$|e\rangle\langle e| + |g\rangle\langle g| = 1$$

In general $\mathcal{H} = \{|e_1\rangle, |e_2\rangle, \ldots |e_n\rangle\}$

elements of the basis

$$1 = \sum_n |e_n\rangle\langle e_n|$$

Time-dynamics:

$$H|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle \Rightarrow \frac{\partial}{\partial t}|\psi\rangle = -\frac{iH}{\hbar}|\psi\rangle$$

$$\frac{\partial x}{\partial t} = \lambda x \qquad \int \frac{\partial x}{x} = \int \lambda \, dt$$

$$\ln\left(\frac{x(t)}{x(0)}\right) = \lambda t \Rightarrow x(t) = e^{\lambda t} x(0)$$

$$|\psi(t)\rangle = e^{-iHt/\hbar} |\psi(0)\rangle \qquad H^+ = H$$

$$\langle\psi(t)| = \langle\psi(0)| e^{iH^+t/\hbar}$$

$$= \langle\psi(0)| e^{iHt/\hbar}$$

$H|\psi\rangle = E|\psi\rangle$ H has to be hermitian

$$\hat{O} = \hat{A}_1 \hat{A}_2 \hat{A}_3 \cdots \hat{A}_n \qquad \hat{O}^\dagger = \hat{A}_n^\dagger \cdots \hat{A}_3^\dagger \hat{A}_2^\dagger \hat{A}_1^\dagger$$

For any operator

$$\langle \psi(t) | \hat{O} | \psi(t) \rangle = \langle \hat{O}(t) \rangle = \int d^3x \, \psi^*(\vec{x},t)\, \hat{O}\, \psi(\vec{x},t)$$

$$\langle \hat{O}(t) \rangle = \langle \psi(t) | \hat{O} | \psi(t) \rangle$$

$$= \langle \psi(0) | e^{iHt/\hbar} \, \hat{O} \, e^{-iHt/\hbar} | \psi(0) \rangle$$

$$= \langle \psi(0) | \hat{O}(t) | \psi(0) \rangle$$

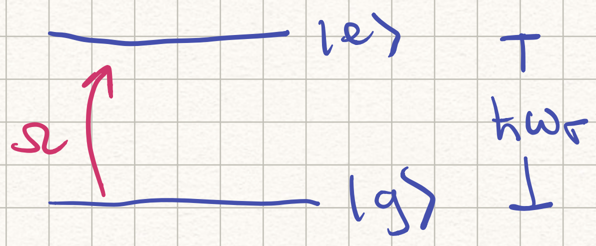
$$\hat{O}(t) = e^{iHt/\hbar} \, \hat{O} \, e^{-iHt/\hbar}$$

If $[\hat{O}, H] = 0 \Rightarrow \hat{O}(t) = \hat{O}(0)$

$$\boxed{i\hbar \, \partial_t \hat{O} = [H, \hat{O}]}$$

Heisenberg Equation

$$H = \hbar\Omega \, |e\rangle\langle g| + \frac{1}{2}\hbar\omega_\sigma \left(|e\rangle\langle e| - |g\rangle\langle g| \right) + \hbar\Omega (|g\rangle\langle e|)$$



$H^\dagger = H$

$$H^\dagger = \hbar\Omega \, |g\rangle\langle e| + \frac{1}{2}\hbar\omega_\sigma \left(|e\rangle\langle e| - |g\rangle\langle g| \right) + \hbar\Omega (|e\rangle\langle g|)$$

Hamiltonian of a 2LS

$$H = \hbar\Omega \left(|e\rangle\langle g| + |g\rangle\langle e| \right) + \hbar\omega_\sigma \left(|e\rangle\langle e| - |g\rangle\langle g| \right)$$

$\sigma = |g\rangle\langle e| \qquad \sigma^\dagger = |e\rangle\langle g|$ Notation

$$H = \frac{1}{2}\hbar\omega_\sigma(\sigma^+\sigma - \sigma\sigma^+) + \hbar\Omega(\sigma^+ + \sigma)$$

$$|e\rangle\langle e| = \sigma^+\sigma = |e\rangle\langle g|g\rangle\langle e|$$

$$|g\rangle\langle g| = \sigma\sigma^+ = |g\rangle\langle e|e\rangle\langle g|$$

$$\sigma^+\sigma + \sigma\sigma^+ = |e\rangle\langle e| + |g\rangle\langle g| = 1$$

$$\sigma\sigma^+ = 1 - \sigma^+\sigma$$

$$\sigma^+\sigma - \sigma\sigma^+ = \sigma^+\sigma - (1 - \sigma^+\sigma)$$

$$= 2\sigma^+\sigma - 1$$

$$H = \frac{1}{2}\hbar\omega_\sigma(2\sigma^+\sigma - 1) + \hbar\Omega(\sigma^+ + \sigma)$$

$$\boxed{H = \hbar\omega_\sigma \cdot \sigma^+\sigma + \hbar\Omega(\sigma^+ + \sigma)}$$

$$|e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |g\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|e\rangle\langle g| = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \sigma^+$$

$$|g\rangle\langle e| = \sigma = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\sigma^+\sigma = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H = \begin{pmatrix} 0 & 0 \\ 0 & \hbar\omega_\sigma \end{pmatrix} + \hbar\Omega\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \hbar\Omega \\ \hbar\Omega & \hbar\omega_\sigma \end{pmatrix}$$

$$\boxed{\begin{aligned} |\psi(t)\rangle &= e^{-iHt/\hbar}\,|\psi(0)\rangle \\ |\psi(0)\rangle &= C_g\,|g\rangle + C_e\,|e\rangle \end{aligned}}$$

Homework.

$$|\psi(t)\rangle = C_g(t)\,|g\rangle + C_e(t)\,|e\rangle$$

Tutorial

Terry Tao → Quanta Mag.

Sonification of Q. correlations

Collect data from people

GDPR: General data protection regulation

- Consent that you keep and process info.
- Sign an informing consent
- which info. you'll be collecting
- How processing it
  storing it
  protecting it
- They can opt-out at any time

→ Check that with GDPR