Semiconductors

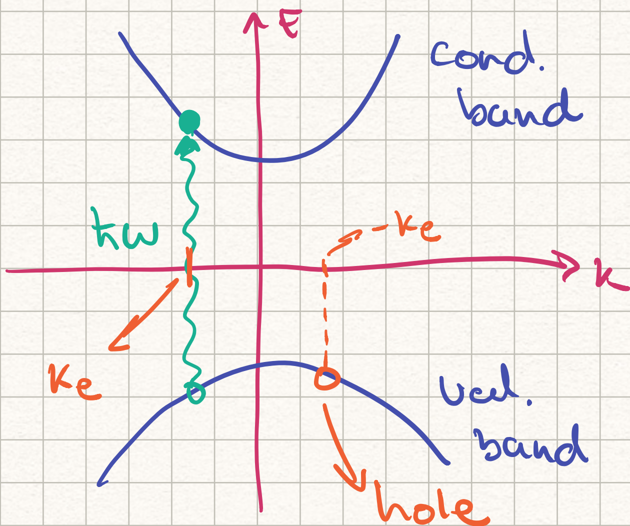
Holes

Vacant orbital in filled bands

They also contribute to the conductivity

Vacancies in a band are called holes

1. $k_e = -k_h$: conservation of momentum



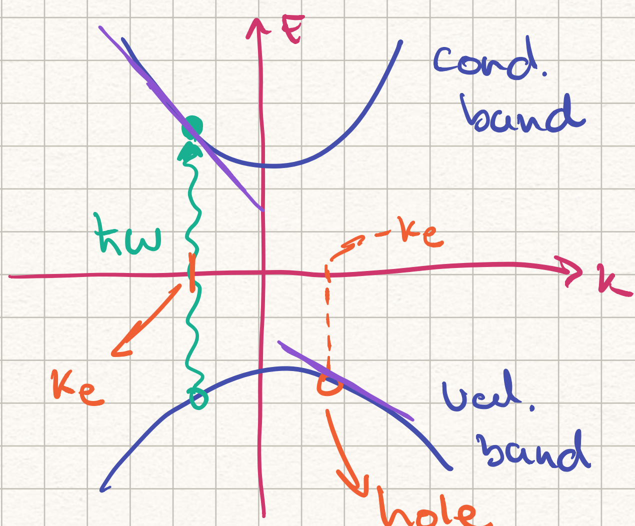
The hole is an alternate description of a band with one missing e^-.

We can say that the hole has momentum $-k_e$, or that the band with one e^- missing has momentum $-k_e$

2. $\varepsilon_h(k_h) = -\varepsilon_e(k_e)$

$$\varepsilon_e(k_e) = \varepsilon_e(-k_e) = -\varepsilon_h(-k_e) = -\varepsilon_h(k_h)$$

3. $\vec{v}_e = \vec{v}_h$



$$\vec{v} = \frac{1}{\hbar} \vec{\nabla}_k \varepsilon$$

$$\vec{v}_e = \frac{1}{\hbar} \vec{\nabla}_k \varepsilon_e(k_e)$$

$$\vec{v}_h = \frac{1}{\hbar} \vec{\nabla}_k \varepsilon_h(k_h)$$

$$= \frac{1}{\hbar} \vec{\nabla}_k \varepsilon_e(k_e) = \vec{v}_e$$

4. $m_h = -m_e$

$E = \frac{\hbar^2 k^2}{2m}$

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m} \Rightarrow m = \frac{\hbar^2}{d^2E/dk^2}$$

$$m_e = \frac{\hbar^2}{d^2\varepsilon_e/dk^2} \qquad m_h = \frac{\hbar^2}{d^2\varepsilon_h/dk^2}$$

$$m_e = -m_h$$

$$\vec{F} = m\vec{a} \qquad \vec{F} \rightarrow \qquad \leftarrow \vec{a}$$

If we have a negative mass

5. $\hbar \dfrac{dk_h}{dt} = \vec{F} = e\left(\vec{E} + \vec{v}_h \times \vec{B}\right)$

$$\hbar \frac{dk_e}{dt} = -e\left(\vec{E} + \vec{v}_e \times \vec{B}\right)$$

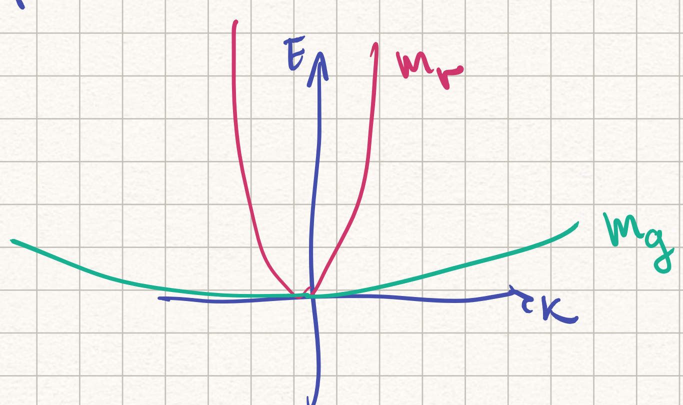
Using $k_e = -k_h$ and $\vec{v}_e = \vec{v}_h$

$$\hbar \frac{dk_h}{dt} = e\left(\vec{E} + \vec{v}_h \times \vec{B}\right)$$

Effective mass

$\varepsilon = \dfrac{\hbar^2 k^2}{2m}$ $\qquad k^2$ determines the curvature

We could also say that $1/m$ is what determines the curvature: how narrow/wide the parabola is depends on m.



$$\boxed{m_r < m_g}$$

If $E_g \ll \lambda$ $\left\{ \lambda = \frac{\hbar^2}{2m}\left(\frac{1}{2}G\right)^2 \right\}$ energy of free electrons

$\Rightarrow$ the curvature is enhanced by a factor λ / E_g

$$E_{k,\pm} = E_\pm + \frac{\hbar^2 k^2}{2m}\left(1 \pm \frac{2\lambda}{U}\right) \quad \text{valid } |U| \gg \lambda$$

$$E_{k,+} = E_c + \frac{\hbar^2 k^2}{2m_e} \quad \text{with } E_c = E_+$$

$$\frac{1}{m_e} = \frac{1}{m}\left(1 + \frac{2\lambda}{U}\right) \Rightarrow \frac{m_e}{m} = \frac{1}{1 + 2\lambda/U}$$

In the valence band

$$E_k = E_v - \frac{\hbar^2 k^2}{2m_h}$$

$$\frac{-1}{m_h} = \frac{1}{m}\left(1 - \frac{2\lambda}{U}\right) \Rightarrow \frac{m_h}{m} = \frac{1}{\frac{2\lambda}{U} - 1}$$

$\rightarrow$ The weight of the crystal doesn't chang

$\rightarrow$ 2nd law of Newton is still valid for the crystal as a whole (e^- and holes)

$\rightarrow$ Apply $\vec{F} \Rightarrow e^-$ accelerates as if it has mass m_e.

$$v_g = \frac{1}{\hbar}\frac{\partial E}{\partial k} \qquad a = \frac{dv_g}{dt} = \frac{1}{\hbar}\frac{\partial^2 E}{\partial k \partial t}$$

$$= \frac{1}{\hbar}\frac{\partial^2 E}{\partial k^2}\frac{\partial k}{\partial t}$$

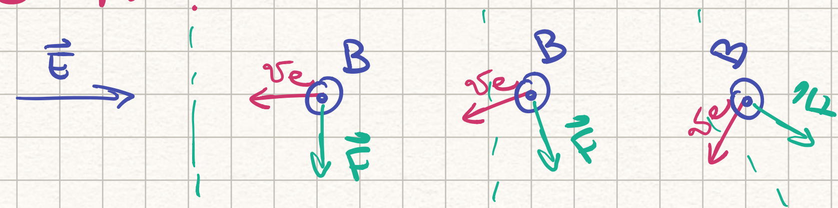
but we know $\vec{F} = \hbar \frac{d\vec{k}}{dt}$

$$a = \frac{1}{\hbar}\frac{\partial^2 E}{\partial k^2}\left(\frac{F}{\hbar}\right) = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2} F = \frac{1}{m^*} F$$

$$\boxed{\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2}}$$ the effective inertial mass

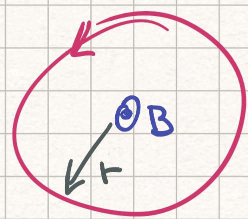
How do we measure m^*?

$$\varepsilon = \frac{\hbar^2 k^2}{2m^*}$$



$$\hbar \frac{dk}{dt} = \vec{F}$$

$$-evB = -\frac{m^* v^2}{r}$$



→ Circular trajectory

$$r = \frac{m^* v}{eB}$$

trajectory has $L = 2\pi r$

frequency $= \frac{v}{L} = \frac{v}{2\pi r} = \frac{v e B}{2\pi m^* v} = \frac{eB}{2\pi m^*} = f$

$$\boxed{\omega = 2\pi f = \frac{eB}{m^*}}$$ cyclotron frequency

In semiconductors $\lambda \sim 10^{-3} - 10^{-2}$ m : microwaves