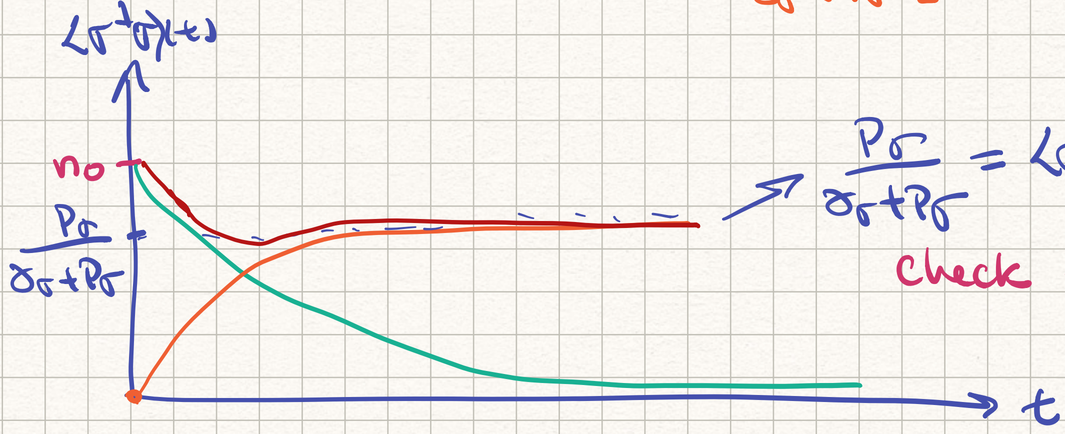
$$\partial_t \langle \sigma^\dagger \sigma \rangle = P_\sigma - (\gamma_\sigma + P_\sigma)\langle \sigma^\dagger \sigma \rangle$$

$$\partial_t X(t) = P_\sigma - (\gamma_\sigma + P_\sigma) X(t)$$

Inhomogeneous diff. eq.

$$X(t) = X(0)\, e^{P_\sigma t / X(t)}\, e^{-(\gamma_\sigma + P_\sigma)t}$$

$$\langle \sigma^\dagger \sigma \rangle(t) = n_0\, e^{-(\gamma_\sigma + P_\sigma)t} + \frac{P_\sigma}{\gamma_\sigma + P_\sigma}\left[1 - e^{-(\gamma_\sigma + P_\sigma)t}\right]$$



$$\frac{P_\sigma}{\gamma_\sigma + P_\sigma} = \langle \sigma^\dagger \sigma \rangle_{ss}$$ steady state

Check that $\langle \sigma^\dagger \sigma \rangle_{ss} \leq 1$

With $\Omega \neq 0$ (Coherent excitation)

$$\partial_t \langle \sigma^\dagger \sigma \rangle = i\Omega\left(\langle \sigma \rangle - \langle \sigma^\dagger \rangle\right) - \gamma_\sigma \langle \sigma^\dagger \sigma \rangle$$

$$\partial_t \langle \sigma \rangle = 2i\Omega \langle \sigma^\dagger \sigma \rangle - i\omega_\sigma \langle \sigma \rangle - i\Omega - \frac{\gamma_\sigma}{2}\langle \sigma \rangle$$

$$\partial_t \langle \sigma^\dagger \rangle = -2i\Omega \langle \sigma^\dagger \sigma \rangle + i\omega_\sigma \langle \sigma^\dagger \rangle + i\Omega - \frac{\gamma_\sigma}{2}\langle \sigma^\dagger \rangle$$

$$\partial_t \vec{v} = M \vec{v} \quad \Rightarrow \quad \vec{v}(t) = e^{Mt}\,\vec{v}(0)$$

$$\partial_t \begin{bmatrix} 1 \\ \langle \sigma \rangle \\ \langle \sigma^\dagger \rangle \\ \langle \sigma^\dagger \sigma \rangle \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -i\Omega & -i\omega_\sigma - \frac{\gamma_\sigma}{2} & 0 & 2i\Omega \\ i\Omega & 0 & i\omega_\sigma - \frac{\gamma_\sigma}{2} & -2i\Omega \\ 0 & i\Omega & -i\Omega & -\gamma_\sigma \end{bmatrix} \begin{bmatrix} 1 \\ \langle \sigma \rangle \\ \langle \sigma^\dagger \rangle \\ \langle \sigma^\dagger \sigma \rangle \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -i\Omega & -i\omega_\sigma - \frac{\gamma_\sigma}{2} & 0 & 2i\Omega \\ i\Omega & 0 & i\omega_\sigma - \frac{\gamma_\sigma}{2} & -2i\Omega \\ 0 & i\Omega & -i\Omega & -\gamma_\sigma \end{bmatrix}$$

Exponential of a matrix

$$e^{Mt} = \sum_{k=0}^{\infty} \frac{(Mt)^k}{k!}$$

$$H = UDU^{-1} \quad \Rightarrow \quad e^{H} = U e^{D} U^{-1}$$

U:

- Find the eigenvectors of H: $\{\vec{w}_1, \vec{w}_2 \dots, \vec{w}_k\}$

$$U \equiv \begin{pmatrix} w_1 & w_2 & w_3 & \dots & w_k \end{pmatrix}^T$$

$$U = \begin{pmatrix} w_{11} & w_{21} & \dots \\ w_{12} & w_{22} & \\ w_{13} & w_{23} & \\ \vdots & \vdots & \\ w_{1k} & w_{2k} & \end{pmatrix}^T$$

$$\langle \sigma^\dagger \sigma \rangle(t) = \frac{4\Omega^2}{\gamma_\sigma^2 + 8\Omega^2} + \frac{\Omega^2}{\gamma_\sigma^2 + 8\Omega^2} \frac{1}{R} \left\{ \left\{ 12\gamma_\sigma \sin\left(\frac{Rt}{4}\right) - 4R\cos\left(\frac{Rt}{4}\right) \right\} e^{-3\gamma_\sigma t/4} \right.$$

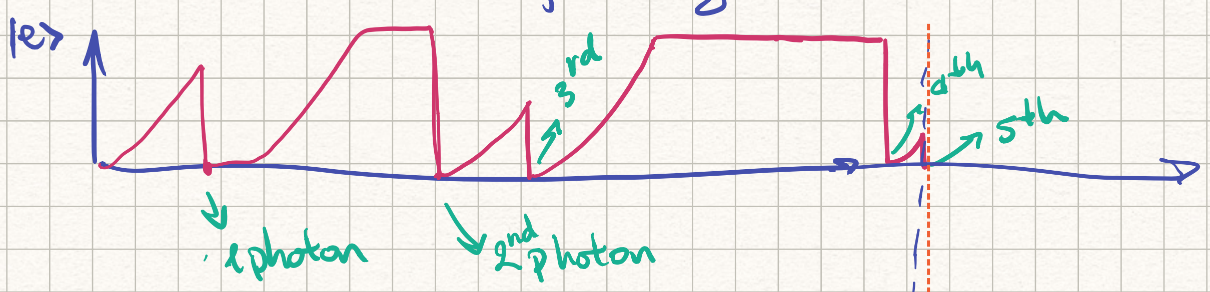
$$R = \sqrt{64\Omega^2 - \gamma_\sigma^2}$$

$$\langle \sigma^\dagger \sigma \rangle_{ss} = \frac{4\Omega^2}{\gamma_\sigma^2 + 8\Omega^2}$$

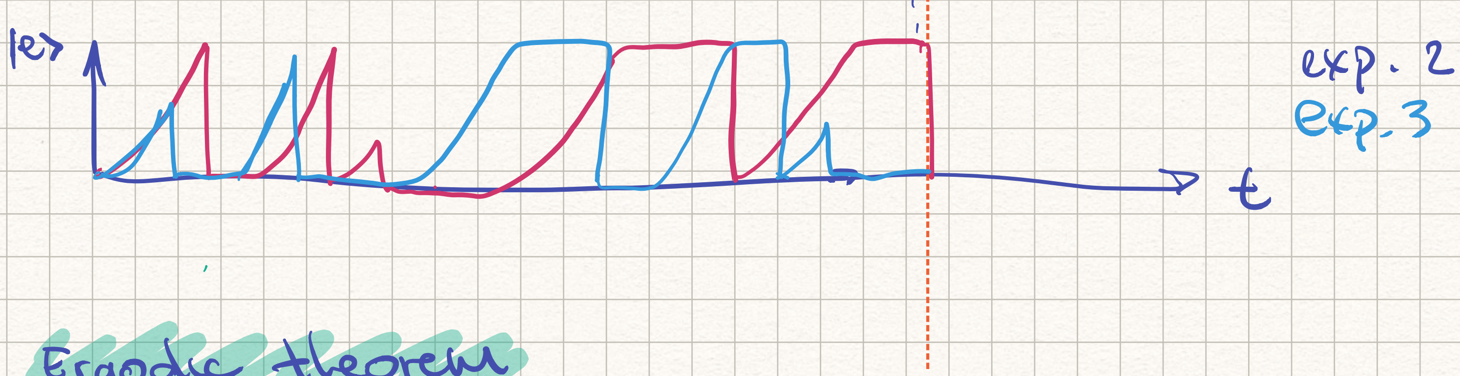
check that $\langle \sigma^\dagger \sigma \rangle_{ss} \leq 0.5$

$\langle \hat{o} \rangle$: Mean value of $\hat{o}$

1. Quantum trajectory



exp. 1



Ergodic theorem

- $\langle \hat{O} \rangle_{conf} = \langle \hat{O} \rangle_T$

→ Real the Mølner et al. paper 1-2 pages

$$\partial_t \rho = \frac{i}{\hbar}[\rho, H] + \frac{1}{2} \sum_k \mathcal{L}_{L_k} \rho$$ Master eq.

$|\phi(t)\rangle$ + q. jumps