Quantum jumps

$$\partial_t \rho = \frac{i}{\hbar}[\rho, H] + \frac{1}{2}\sum_k \mathcal{L}_{L_k}\rho$$

$$\mathcal{L}_{L_k}\rho = 2L_k\rho L_k^\dagger - L_k^\dagger L_k\rho - \rho L_k^\dagger L_k$$

Dissipative processes

Define $\tilde{H} = H - \frac{i\hbar}{2}\sum_k L_k^\dagger L_k$ $\qquad \tilde{H}^\dagger \neq \tilde{H}$

$$\tilde{H}^\dagger = H^\dagger - \left(\frac{i\hbar}{2}\sum_k L_k^\dagger L_k\right)^\dagger$$

$$= H - \left(-\frac{i\hbar}{2}\sum_k L_k^\dagger L_k\right)$$

$$= H + \frac{i\hbar}{2}\sum_k L_k^\dagger L_k$$

$|\phi(t)\rangle$ evolves with $\tilde{H}$

$$i\hbar\,\partial_t|\phi(t)\rangle = \tilde{H}|\phi(t)\rangle$$

$$\partial_t|\phi(t)\rangle = -\frac{i\tilde{H}}{\hbar}|\phi(t)\rangle$$

$$|\phi^{(1)}(t+\delta t)\rangle = e^{-i\tilde{H}\delta t/\hbar}|\phi(t)\rangle$$

If $\delta t \ll 1 \Rightarrow e^{-i\tilde{H}\delta t/\hbar} = 1 - \frac{i\tilde{H}\delta t}{\hbar} + \mathcal{O}(\delta t^2)$

$$|\phi^{(1)}(t+\delta t)\rangle = \left(1 - \frac{i\tilde{H}\delta t}{\hbar}\right)|\phi(t)\rangle$$

$$\langle\phi^{(1)}(t+\delta t)| = \langle\phi(t)|\left(1 + \frac{i\tilde{H}^\dagger\delta t}{\hbar}\right)$$

$$\langle\phi^{(1)}(t+\delta t)|\phi^{(1)}(t+\delta t)\rangle \overset{?}{=} 1$$

$$\langle \phi(t) | \left(1 + \frac{i\tilde{H}^{\dagger}\delta t}{\hbar}\right)\left(1 - \frac{i\tilde{H}\delta t}{\hbar}\right) | \phi(t) \rangle$$

$$\left(1 + \frac{i\tilde{H}^{\dagger}\delta t}{\hbar}\right)\left(1 - \frac{i\tilde{H}\delta t}{\hbar}\right) = 1 - \frac{i\tilde{H}\delta t}{\hbar} + \frac{i\tilde{H}^{\dagger}\delta t}{\hbar} - \cancel{i^2 \frac{\delta t^2}{\hbar^2} \tilde{H}^{\dagger}\tilde{H}} \to 0$$

$$= 1 - \frac{i\delta t}{\hbar}\left(\tilde{H} - \tilde{H}^{\dagger}\right)$$

$$\langle \phi(t) | \left[1 - \frac{i\delta t}{\hbar}\left(\tilde{H} - \tilde{H}^{\dagger}\right)\right] | \phi(t) \rangle = \langle \phi(t) | \phi(t) \rangle$$

$$- \frac{i\delta t}{\hbar} \langle \phi(t) | \left(\tilde{H} - \tilde{H}^{\dagger}\right) | \phi(t) \rangle$$

$$= 1 - \frac{i\delta t}{\hbar} \langle \phi(t) | \left(\tilde{H} - \tilde{H}^{\dagger}\right) | \phi(t) \rangle$$

$$\tilde{H} - \tilde{H}^{\dagger} = H - \frac{i\hbar}{2} \sum_k L_k^{\dagger} L_k - \left(H - \frac{i\hbar}{2} \sum_k L_k^{\dagger} L_k\right)^{\dagger}$$

$$= H - \frac{i\hbar}{2} \sum_k L_k^{\dagger} L_k - H - \frac{i\hbar}{2} \sum_k L_k^{\dagger} L_k$$

$$= -i\hbar \sum_k L_k^{\dagger} L_k$$

$$\langle \phi(t) | \tilde{H} - \tilde{H}^{\dagger} | \phi(t) \rangle = -i\hbar \sum_k \langle \phi(t) | L_k^{\dagger} L_k | \phi(t) \rangle$$

$$\langle \phi^{(1)}(t+\delta t) | \phi^{(1)}(t+\delta t) \rangle = 1 - \frac{i\delta t}{\hbar}(-i\hbar) \sum_k \langle \phi(t) | L_k^{\dagger} L_k | \phi(t) \rangle$$

$$= 1 - \delta t \sum_k \langle \phi(t) | L_k^{\dagger} L_k | \phi(t) \rangle$$

$$= 1 - \delta p$$

$\langle \sigma^{\dagger} \sigma \rangle$: excitation occupation population

$$\frac{\delta p_k}{\delta p} = \Pi_k \qquad \delta p_k = \delta t \langle \phi(t) | L_k^{\dagger} L_k | \phi(t) \rangle$$

$$\sum_k \delta P_k = \delta P \;\Rightarrow\; \sum_k \Pi_k = 1$$

Π_k: Probability of each L_k inducing a Q. Jump.

$$|\phi(t+\delta t)\rangle = \frac{|\phi^{(1)}(t+\delta t)\rangle}{\sqrt{1-\delta p}}$$

2. Quantum jump

- Stochastic processes $\Rightarrow$ Random

ϵ : Random number

$$\delta P_k > \epsilon \;\Rightarrow\; |\phi^{(2)}(t+\delta t)\rangle = L_k |\phi(t)\rangle$$

$$\langle \phi^{(2)}(t+\delta t) | \phi^{(2)}(t+\delta t)\rangle = \langle \phi(t)| L_k^{+} L_k |\phi(t)\rangle$$

$$|\phi(t+\delta t)\rangle = \frac{L_k |\phi(t)\rangle}{\sqrt{\langle \phi(t)| L_k^{+} L_k |\phi(t)\rangle}} \qquad \langle \phi(t+\delta t)| = \frac{\langle \phi(t)| L_k^{+}}{\sqrt{\langle \phi(t)| L_k^{+} L_k |\phi(t)\rangle}}$$

Let's check that this methods is consistent with the master equation

$$\sigma(t+\delta t) = \overline{|\phi(t+\delta t)\rangle\langle\phi(t+\delta t)|} - \overline{|\times|}$$

$$= (1-\delta p)\frac{|\phi^{(1)}(t+\delta t)\rangle}{\sqrt{1-\delta p}}\frac{\langle \phi^{(1)}(t+\delta t)|}{\sqrt{1-\delta p}} +$$

$$+ \delta p \sum_k \Pi_k \frac{L_k |\phi(t)\rangle}{\sqrt{\langle \phi(t)| L_k^{+} L_k |\phi(t)\rangle}} \frac{\langle \phi(t)| L_k^{+}}{\sqrt{\langle \phi(t)| L_k^{+} L_k |\phi(t)\rangle}}$$

⊛

$$|\phi^{(1)}(t+\delta t)\rangle\langle\phi^{(1)}(t+\delta t)| = \left(1 - \frac{i\tilde{H}\delta t}{\hbar}\right)|\phi(t)\rangle\langle\phi(t)|\left(1 + \frac{i\tilde{H}^{+}\delta t}{\hbar}\right)$$

$$= |\phi(t)\rangle\langle\phi(t)| - \frac{i\delta t}{\hbar}\tilde{H}|\phi\rangle\langle\phi| + \frac{i\delta t}{\hbar}|\phi(t)\rangle\langle\phi(t)|\tilde{H}^{+}$$

$$= \sigma(t) - \frac{i\delta t}{\hbar}\left(\tilde{H}\sigma(t) - \sigma(t)\tilde{H}^{+}\right)$$

$$\tilde{H} = H - \frac{i\hbar}{2}\sum_k' L_k^\dagger L_k \qquad \tilde{H}^\dagger = H + \frac{i\hbar}{2}\sum_k' L_k^\dagger L_k$$

$$\tilde{H}\sigma(t) = H\sigma(t) - \frac{i\hbar}{2}\sum_k' L_k^\dagger L_k \sigma(t)$$

$$\sigma(t)\tilde{H}^\dagger = \sigma(t)H + \frac{i\hbar}{2}\sigma(t)\sum_k' L_k^\dagger L_k$$

$$\tilde{H}\sigma(t) - \sigma(t)\tilde{H}^\dagger = H\sigma(t) - \sigma(t)H - \frac{i\hbar}{2}\sum_k'\left(L_k^\dagger L_k\sigma(t) + \sigma(t)L_k^\dagger L_k\right)$$

$$= \sigma(t) - \frac{i\delta t}{\hbar}\left\{[H,\sigma(t)] - \frac{i\hbar}{2}\sum_k'\left(L_k^\dagger L_k\sigma(t) + \sigma(t)L_k^\dagger L_k\right)\right\}$$

$$= \sigma(t) + \frac{i}{\hbar}\delta t[\sigma(t), H] - \frac{\delta t}{2}\sum_k'\left(L_k^\dagger L_k\sigma(t) + \sigma(t)L_k^\dagger L_k\right)$$

$$\otimes$$

$$\delta p \sum_k' \pi_k \frac{L_k|\phi(t)\rangle}{\sqrt{\langle\phi(t)|L_k^\dagger L_k|\phi(t)\rangle}} \frac{\langle\phi(t)|L_k^\dagger}{\sqrt{\langle\phi(t)|L_k^\dagger L_k|\phi(t)\rangle}} =$$

$$\delta p \sum_k' \frac{L_k\sigma(t)L_k^\dagger}{\langle\phi(t)|L_k^\dagger L_k|\phi(t)\rangle}\frac{\delta t}{\delta p}\langle\phi(t)|L_k^\dagger L_k|\phi(t)\rangle$$

$$\pi_k = \frac{\delta p_k}{\delta p} = \frac{\delta t}{\delta p}\langle\phi(t)|L_k^\dagger L_k|\phi(t)\rangle$$

$$= \delta t \sum_k L_k\sigma(t)L_k^\dagger$$

$$\bar{\sigma}(t+\delta t) = \sigma(t) + \frac{i}{\hbar}\delta t[\sigma(t), H] - \frac{\delta t}{2}\sum_k'\left(L_k^\dagger L_k\sigma(t) + \sigma(t)L_k^\dagger L_k\right) + \delta t\sum_k' L_k\sigma(t)L_k^\dagger$$

$$\sigma(t+\delta t) - \sigma(t) = \frac{i}{\hbar}\delta t\,[\sigma(t), H] - \frac{\delta t}{2}\sum_k \left(L_k^\dagger L_k \sigma(t) + \sigma(t) L_k^\dagger L_k\right)$$

$$+ \delta t \sum_k L_k \sigma(t) L_k^\dagger$$

$$\frac{\sigma(t+\delta t) - \sigma(t)}{\delta t} = \frac{i}{\hbar}[\sigma(t), H] + \frac{1}{2}\sum_k \left(2 L_k \sigma(t) L_k^\dagger - L_k^\dagger L_k \sigma(t) - \sigma(t) L_k^\dagger L_k\right)$$

$$\lim_{\delta t \to 0} \frac{\sigma(t+\delta t) - \sigma(t)}{\delta t} = \partial_t \sigma(t)$$

$$\boxed{\partial_t \sigma(t) = \frac{i}{\hbar}[\sigma(t), H] + \frac{1}{2}\sum_k \left(2 L_k \sigma(t) L_k^\dagger - L_k^\dagger L_k \sigma(t) - \sigma(t) L_k^\dagger L_k\right)}$$

Master eq. for $\sigma(t)$.