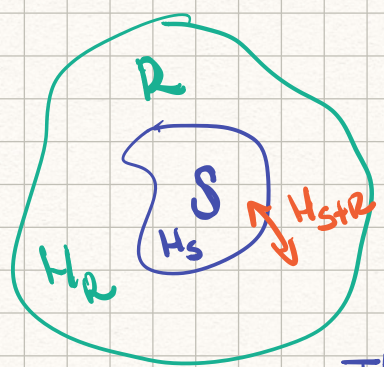
Dissipation

Our Q-system is in contact with the rest of the universe

$$H_T = H_S + H_R + H_{S+R}$$

H_S: Hamil. of the system
H_R: Haml. of the reservoir
H_{S+R}: Haml. of the coupling between system and reservoir



- we should find $|\psi\rangle$ that satisfies schrödinger eq. with H_T.

The number of degrees of freedom and the problem becomes untractable.

→ we're interested in the degrees of freedom of the system

Let's assume that we know the density matrix of the entire system → $\Delta(t)$

$\Delta(t)$: density matrix of $S \otimes R$

we're interested in the "reduced" matrix obtained by "tracing out" the degrees of freedom of the reservoir

$$\rho(t) = Tr_R[\Delta(t)]$$: $\rho(t)$ is the partial trace of Δ w.r.t. R.

How do we obtain a partial trace.

$$\Delta = \sum'_{\substack{\nu,\mu \\ n,m}} C^{\nu,\mu}_{n,m} |\nu,n\rangle\langle\mu,m|$$

In the example of last class:

$$\rho = |c_g|^2 |g\rangle\langle g| + c_e^* c_g |g\rangle\langle e| + c_e c_g^* |e\rangle\langle g| + |c_e|^2 |e\rangle\langle e|$$

$$Tr[\rho] = \langle g|\rho|g\rangle + \langle e|\rho|e\rangle = \sum'_{\nu=\{g,e\}} \langle\nu|\rho|\nu\rangle$$

$$= |c_g|^2 + |c_e|^2$$

we define ρ through its matrix elements:

$$\langle i|\rho|j\rangle = \sum_{\varepsilon} \langle i,\varepsilon|\Delta|j,\varepsilon\rangle$$

$$= \sum_{\varepsilon} \sum'_{\substack{\mu,\nu \\ n,m}} C^{\nu,\mu}_{n,m} \langle i,\varepsilon|\nu,n\rangle\langle\mu,m|j,\varepsilon\rangle$$

$$\langle\mu,m|j,\varepsilon\rangle = \delta_{\mu j}\,\delta_{m,\varepsilon}$$

$$= \sum_{\varepsilon} \sum_{\substack{\mu,\nu \\ n,m}} C^{\nu,\mu}_{n,m} \delta_{i\nu}\,\delta_{\varepsilon n}\,\delta_{\mu j}\,\delta_{m,\varepsilon}$$

$$= \sum_{\varepsilon} \sum'_{n,m} C^{ij}_{nm}\, \delta_{\varepsilon n}\,\delta_{m\varepsilon}$$

δ_{en}
δ_{mn}

$$= \sum'_{n} C^{ij}_{nn}$$

Mean value of $\hat{o}$ of the system

$$\langle\hat{o}\rangle(t) = Tr_{S\otimes R}[\hat{o}\,\Delta(t)] = Tr_S[\hat{o}\, Tr_R[\Delta(t)]]$$

$$= Tr_S[\hat{o}\,\rho(t)]$$

The goal is to obtain the eq. of motion of $\rho(t)$ with the properties of the reservoir entering as parameters

The Master equation (in the Lindblad form)

$$\partial_t \rho = \frac{i}{\hbar}[\rho, H] + \frac{1}{2}\sum_k \mathcal{L}_{L_k}\rho$$

$\mathcal{L}_{L_k}$: Liouville operators

$$\mathcal{L}_{L_k}\rho = \left(2 L_k^{+} \rho L_k - L_k^{+} L_k \rho - \rho L_k^{+} L_k\right)$$

The operators L_k are commonly referred to as Jump operators

1. Born approximation:

At some $t=0$: $\Delta(0) = \rho(0) R_0$

R_0 : Initial environment operator

If the reservoir is large enough, it is not perturbed by the system $\Delta(t) = \rho(t) R_0$

2. Markov approximation

If Γ_i and Γ_j are operators of the reservoir, their correlations are localized in time

$$\langle \Gamma_i(t)\, \Gamma_j(t') \rangle_R \propto \delta(t-t')$$

Together, they are called the Born-Markov approximation