Time-reversal symetry

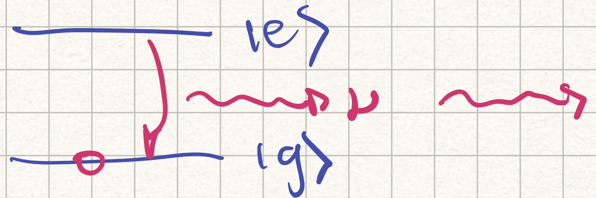
$$|\psi(t+\tau)\rangle = e^{-iH\tau/\hbar}\,|\psi(t)\rangle$$

because H is hermitian $\quad H^{+} = H$

$$|\psi(t)\rangle = e^{iH\tau/\hbar}\,|\psi(t+\tau)\rangle$$

At some point there will be a decay from the excited state (high energy state) to the ground state

- Decay is poissonian process



Process is not reversible
→ It's not described by a Hamiltonian

$|\psi(t)\rangle$ given that we know $|\psi(0)\rangle$

$$\partial_t |\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle \qquad \text{Schrödinger Eq.}$$

$$(|\psi\rangle)^{+} = \langle\psi|$$

$$(\partial_t|\psi\rangle)^{+} = \partial_t \langle\psi| = \left(-\frac{i}{\hbar} H |\psi\rangle\right)^{+} = \frac{i}{\hbar}\langle\psi| H^{+}$$

$$= \frac{i}{\hbar}\langle\psi| H$$

Let's consider $|\psi\rangle\langle\psi|$

$$\partial_t (|\psi\rangle\langle\psi|) = (\partial_t |\psi\rangle)\langle\psi| + |\psi\rangle(\partial_t \langle\psi|)$$

$$= \frac{-i}{\hbar} H|\psi\rangle\langle\psi| + \frac{i}{\hbar}|\psi\rangle\langle\psi| H$$

$$= \frac{i}{\hbar}\left(|\psi\rangle\langle\psi| H - H|\psi\rangle\langle\psi|\right)$$

This is a commutator

we define $\rho = |\psi\rangle\langle\psi|$

$$\boxed{\partial_t \rho = \frac{i}{\hbar}(\rho H - H\rho) = \frac{i}{\hbar}[\rho, H]}$$

Liouville-von Neumann Equation

ρ: density matrix $\Rightarrow$ generalizes the $|\psi\rangle$

Let's consider the 2LS

$$|\psi(t)\rangle = C_g(t)|g\rangle + C_e(t)|e\rangle$$

$$\langle\psi(t)| = C_g^*(t)\langle g| + C_e^*(t)\langle e|$$

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|$$

$$= (C_g(t)|g\rangle + C_e(t)|e\rangle) \otimes (C_g^*(t)\langle g| + C_e^*(t)\langle e|)$$

$$= C_g(t)C_g^*(t)|g\rangle\langle g| + C_g(t)C_e^*(t)|g\rangle\langle e|$$

$$= C_e(t)C_g^*(t)|e\rangle\langle g| + C_e(t)C_e^*(t)|e\rangle\langle e|$$

we can write $\rho(t)$ as a matrix

$$\begin{array}{c c c}
 & |g\rangle & |e\rangle & \\
\rho(t) = & \left(\begin{matrix} |C_g(t)|^2 & C_e(t)C_g^*(t) \\ C_g(t)C_e^*(t) & |C_e(t)|^2 \end{matrix}\right) & \begin{matrix} \langle g| \\ \langle e| \end{matrix}
\end{array}$$

$|g\rangle\langle g|$

$|e\rangle\langle g|$

$|g\rangle\langle e|$

$\langle \sigma^+\sigma \rangle(t) = |C_e(t)|^2 \qquad 1 - \langle \sigma^+\sigma \rangle(t) = |C_g(t)|^2$

$\langle \sigma \rangle(t) = C_e(t) C_g^*(t) \qquad \langle \sigma^+ \rangle = C_e^*(t) C_g(t)$ Check

$$\rho(t) = \begin{pmatrix} 1 - \langle \sigma^+\sigma \rangle(t) & \langle \sigma \rangle(t) \\ \langle \sigma^+ \rangle(t) & \langle \sigma^+\sigma \rangle(t) \end{pmatrix}$$

1. $\mathrm{Tr}[\rho(t)] = 1$
2. $\rho(t) = \rho^+(t)$: $\rho(t)$ is Hermitian
   ↳ Eigenvalues of $\rho(t)$ are real numbers
3. $\langle \sigma \rangle \langle \sigma^+ \rangle \leq (1 - \langle \sigma^+\sigma \rangle) \langle \sigma^+\sigma \rangle$

$$\langle \sigma \rangle \langle \sigma^+ \rangle = C_g^* C_e C_g C_e^* = |C_g(t)|^2 |C_e(t)|^2$$
$$= (1 - \langle \sigma^+\sigma \rangle) \langle \sigma^+\sigma \rangle$$

when $\langle \sigma \rangle \langle \sigma^+ \rangle = (1 - \langle \sigma^+\sigma \rangle) \langle \sigma^+\sigma \rangle$
we say that $\rho(t)$ is a pure state
There is a $|\psi\rangle \Rightarrow \rho = |\psi\rangle\langle\psi|$

when $\langle \sigma \rangle \langle \sigma^+ \rangle < (1 - \langle \sigma^+\sigma \rangle) \langle \sigma^+\sigma \rangle$
we say that $\rho(t)$ is a mixed state
there's no $|\psi\rangle \Rightarrow \rho = |\psi\rangle\langle\psi|$

Let's consider

$$\rho(t) = \begin{pmatrix} \sin^2(\omega t) & 0 \\ 0 & \cos^2(\omega t) \end{pmatrix}$$ This state is completely mixed

Processes of decoherence
↳ take pure states and convert them into mixed states

- Interaction of the quantum system with its environment
  - → temperature
  - → phonons
  - → EM fields ...

Homework:

$$\rho(t) = \begin{pmatrix} \alpha^2 & \lambda \alpha^* \beta \\ \lambda^* \alpha \beta^* & \beta^2 \end{pmatrix}$$

- What properties must α, β satisfy?
- For which values of λ you can get the $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$?