Rabi Oscillations

$|\psi\rangle = \alpha(t)\,|-\rangle + \beta(t)\,|+\rangle \qquad \alpha,\beta \in \mathbb{C}$

$|\alpha(t)|^2 + |\beta(t)|^2 = 1$

$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad$ eigenstates of σ_z

excited state — ground state

Ladder operator

$\sigma = |-\rangle\langle+| \qquad \sigma|-\rangle = |-\rangle\underbrace{\langle+|-\rangle}_{0} = 0$

$\sigma^\dagger = |+\rangle\langle-| \qquad \sigma|+\rangle = |-\rangle\underbrace{\langle+|+\rangle}_{1} = |-\rangle$

$\sigma^\dagger|+\rangle = |+\rangle\langle-|+\rangle = 0 \qquad \sigma^\dagger|-\rangle = |+\rangle\underbrace{\langle-|-\rangle}_{1} = |+\rangle$

Schrödinger Eq.: $\quad i\hbar\,\partial_t|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$

Free fermion: $\quad \hat{H} = \hbar\omega\,\sigma^\dagger\sigma \qquad \sigma^\dagger\sigma$: number (of particles) operator.

$\sigma^\dagger\sigma = |+\rangle\langle-|-\rangle\langle+| = |+\rangle\langle+|$

$|\psi(t)\rangle = \alpha(t)\,|-\rangle + \beta(t)\,|+\rangle$

$i\hbar\,\partial_t|\psi(t)\rangle = i\hbar\left(\partial_t\alpha(t)\,|-\rangle + \partial_t\beta(t)\,|+\rangle\right)$

$\hat{H}|\psi(t)\rangle = \left(\hbar\omega\,|+\rangle\langle+|\right)\left(\alpha(t)\,|-\rangle + \beta(t)\,|+\rangle\right)$

$= \hbar\omega\,\beta(t)\,|+\rangle$

$i\hbar\left(\partial_t\alpha(t)\,|-\rangle + \partial_t\beta(t)\,|+\rangle\right) = \hbar\omega\,\beta(t)\,|+\rangle$

$i\hbar\,\partial_t\alpha(t) = 0 \qquad \alpha(t) = \alpha_0$

$i\hbar\,\partial_t\beta(t) = \hbar\omega\,\beta(t) \qquad \beta(t) = \beta_0 e^{-i\omega t} \leftarrow$

$|\psi(t)\rangle = \alpha_0\,|-\rangle + \beta_0 e^{-i\omega t}\,|+\rangle \Leftarrow$

$\langle \sigma^\dagger \sigma \rangle$: mean number of particles

$$n_\sigma(t) = \langle \psi(t) | \sigma^\dagger \sigma | \psi(t) \rangle$$

Show that $n_\sigma(t) = |\beta_0|^2$

This $\hat{H}$ conserves the number of particles in the field.

A Fermion driven by a lase.

$$\hat{H} = \hbar\Delta\, \sigma^\dagger\sigma + \hbar\Omega(\sigma^\dagger + \sigma)$$

$\Delta = \omega - \omega_L$ $\quad$ ω: freq. of the fermion

ω_L: freq. of the laser

$\hbar\Omega$: Intensity of laser.

$$i\hbar\, \partial_t |\psi(t)\rangle = i\hbar\left(\partial_t \alpha(t)|-\rangle + \partial_t \beta(t)|+\rangle\right)$$

$$\hat{H}|\psi(t)\rangle = \hbar\Delta\, |+\rangle\langle+|\psi(t)\rangle + \hbar\Omega\left(|+\rangle\langle-|\psi(t)\rangle + |-\rangle\langle+|\psi(t)\rangle\right)$$

$$= \hbar\Delta\,\beta(t)|+\rangle + \hbar\Omega\,\alpha(t)|+\rangle + \hbar\Omega\,\beta(t)|-\rangle$$

$$= \hbar\left[\Delta\beta(t) + \Omega\alpha(t)\right]|+\rangle + \hbar\Omega\beta(t)|-\rangle$$

$$= i\hbar\left(\partial_t\alpha(t)|-\rangle + \partial_t\beta(t)|+\rangle\right)$$

$$i\hbar\,\partial_t\alpha(t) = \hbar\Omega\,\beta(t) \quad \Leftarrow \quad \beta(t) = \frac{i}{\Omega}\partial_t\alpha(t)$$

$$i\hbar\,\partial_t\beta(t) = \hbar\left[\Delta\beta(t) + \Omega\alpha(t)\right]$$

$$\partial_t\left[i\hbar\,\partial_t\alpha(t)\right] = \hbar\Omega\,\partial_t\beta(t)$$

$$i\hbar\,\partial_t^2\alpha(t) = \hbar\Omega\,\partial_t\beta(t)$$

$$= \hbar\Omega\frac{\hbar}{i\hbar}\left[\Delta\beta(t) + \Omega\alpha(t)\right]$$

$$= -i\hbar\Omega\left[\Delta\beta(t) + \Omega\alpha(t)\right]$$

$$i\hbar\,\partial_t^2\alpha(t) = -i\hbar\Omega\left[i\frac{\Delta}{\Omega}\partial_t\alpha(t) + \Omega\alpha(t)\right]$$

$$\partial_t^2 \alpha(t) = -\Omega\left[i\frac{\Delta}{\Omega}\partial_t \alpha(t) + \Omega\,\alpha(t)\right]$$

$$\partial_t^2 \alpha(t) = -i\Delta\,\partial_t \alpha(t) - \Omega^2 \alpha(t)$$

$$\beta(t) = \frac{i}{\Omega}\partial_t \alpha(t)$$

$$\alpha(t) = e^{-i\Delta t/2}\left(C_1 e^{-iRt/2} + C_2 e^{iRt/2}\right)$$

check that this a solution!

$R = \sqrt{\Delta^2 + 4\Omega^2}$: Rabi frequency.

C_1, C_2 constants of integration

$$\frac{i}{\Omega}\partial_t \alpha(t) = \beta(t) = e^{-i\Delta t/2}\left[C_1 e^{-iRt/2}\left(\frac{\Delta+R}{2\Omega}\right) + C_2 e^{iRt/2}\left(\frac{\Delta-R}{2\Omega}\right)\right]$$

$$\alpha(t=0) = \alpha_0 = C_1 + C_2$$

$$\beta(t=0) = \beta_0 = C_1\left(\frac{\Delta+R}{2\Omega}\right) + C_2\left(\frac{\Delta-R}{2\Omega}\right)$$

$$C_1 = \alpha_0\left(\frac{R-\Delta}{2R}\right) + \beta_0\frac{\Omega}{R}$$

$$C_2 = \alpha_0\left(\frac{R+\Delta}{2R}\right) - \beta_0\frac{\Omega}{R}$$

$$\alpha(t) = \frac{e^{i\Delta t/2}}{R}\left[\alpha_0 R\cos\left(\frac{Rt}{2}\right) - i\left(\alpha_0\Delta + 2\beta_0\Omega\right)\sin\left(\frac{Rt}{2}\right)\right]$$

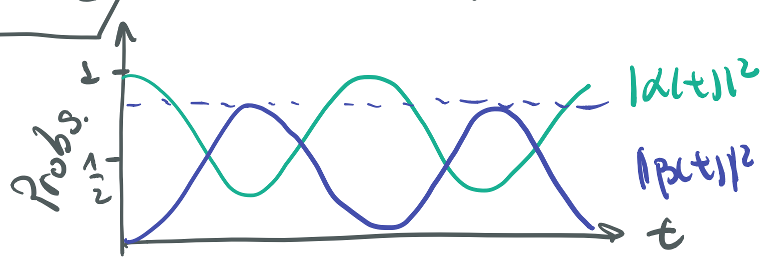
$$\beta(t) = \frac{e^{i\Delta t/2}}{R}\left[\beta_0 R\cos\left(\frac{Rt}{2}\right) + i\left(\beta_0\Delta + 2\alpha_0\Omega\right)\sin\left(\frac{Rt}{2}\right)\right]$$

$$\langle\sigma^\dagger\sigma\rangle = |\beta(t)|^2 =$$

If $\alpha_0 = 1$ (initially in the ground state).

$$n_\sigma(t) = \frac{4\Omega^2}{\Delta^2 + 4\Omega^2}\sin^2\left(\frac{Rt}{2}\right)$$

$\boxed{\Delta = \Omega}$



Go to canvas and see the nice plotting!

$\boxed{\Delta = 0}$ Laser is resonant to the 2LS

$$R = \sqrt{4\Omega^2 + \Delta^2} = 2\Omega$$

$$n_\sigma(t) = \sin^2(\Omega t) \qquad \Omega: \text{freq. of oscillation}$$