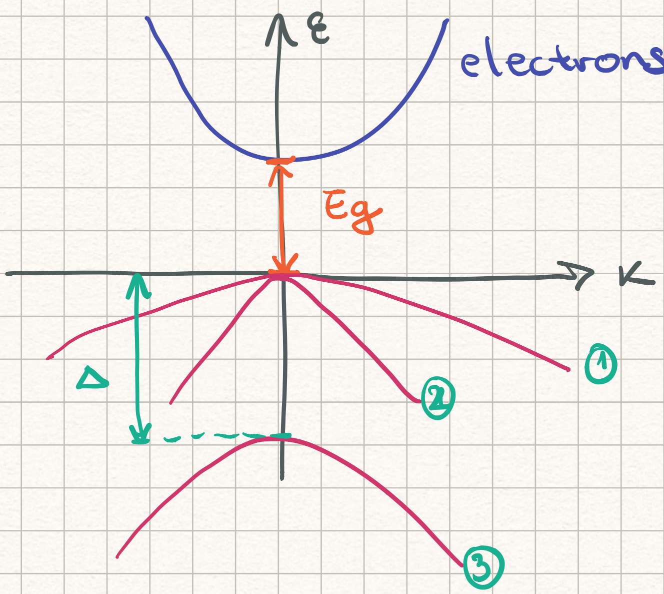
Carrier concentration

For direct gap semiconductors



$$E = E_g + \frac{\hbar^2 k^2}{2m}$$

① Heavy holes
② Light holes
③ Split-off holes

$$\varepsilon_{hh} = -\frac{\hbar^2 k^2}{2m_{hh}} \qquad \varepsilon_{lh} = -\frac{\hbar^2 k^2}{2m_{lh}} \qquad \varepsilon_{soh} = -\Delta - \frac{\hbar^2 k^2}{2m_{soh}}$$

We are assuming that $k^2 = k_x^2 + k_y^2 + k_z^2$

These bands are symetrical in reciprocal space
↳ they are spherical

Δ: spin-orbit coupling splitting } → Relativistic effect

Interaction of the magnetic dipole of an e^- with the EM field of the nucleus because of the motion of the e^- around it (in an orbital)

Silicon and Germanium

The bands are not spherical

$$\varepsilon(\vec{k}) = A k^2 \pm \left[B^2 k^4 + C(k_x^2 k_y^2 + k_y^2 k_z^2 + k_z^2 k_x^2)\right]^{1/2}$$ heavy-light-holes

$$\varepsilon_{soh}(\vec{k}) = -\Delta + A k^2$$ split-off holes

Intrinsic carrier concentration

Simple parabolic bands edges

$$f(\varepsilon) = \frac{1}{\exp\left[\frac{\varepsilon-\mu}{k_BT}\right]+1}$$

μ: Fermi Energy

Chemical potential = Fermi energy

If $\varepsilon - \mu \gg k_BT$

$$f(\varepsilon) \approx \exp\left[-\frac{\varepsilon-\mu}{k_BT}\right] \ll 1$$

$$\varepsilon_k = E_c + \frac{\hbar^2 k^2}{2m_e}$$

E_c: Energy at the conduction band edge

$$D_e(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(\varepsilon - E_c)^{1/2}$$

The energy is referred to the conduction band edge

The concentration of e^- in the conduction band

$$n = \int_{E_c}^{\infty} D_e(\varepsilon) f(\varepsilon)\, d\varepsilon$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}\int_{E_c}^{\infty}(\varepsilon-E_c)^{1/2}\exp\left(-\frac{\varepsilon-\mu}{k_BT}\right)d\varepsilon$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} e^{\mu/k_BT}\underbrace{\int_{E_c}^{\infty}(\varepsilon-E_c)^{1/2}e^{-\varepsilon/k_BT}d\varepsilon}_{\frac{\sqrt{\pi}}{2}(k_BT)^{3/2}e^{-E_c/k_BT}}$$

$$= 2\left(\frac{m_e k_BT}{2\pi\hbar^2}\right)^{3/2}\exp\left[\frac{\mu-E_c}{k_BT}\right]$$

Concentration of holes

$f_h(\varepsilon) = 1 - f_e(\varepsilon)$: a hole is the absence of e^-

$$= \frac{1}{\exp\left[\frac{\mu-\varepsilon}{k_BT}\right]+1}$$

$$D_h(\epsilon) = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(E_v - \epsilon)^{1/2}$$

$$p = \int_{-\infty}^{E_v} D_h(\epsilon) f_h(\epsilon)\, d\epsilon = 2\left(\frac{m_h k_B T}{2\pi \hbar^2}\right)^{3/2} \exp\left[\frac{E_v - \mu}{k_B T}\right]$$

Thus

$$np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} \exp\left[\frac{-E_c + E_v}{k_B T}\right]$$

$$E_c - E_v = E_g \Rightarrow -E_c + E_v = -E_g$$

$$\boxed{np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} \exp\left(-E_g / k_B T\right)}$$

Independent of the Fermi level

At some T the np value is constant.

- Photons create electron-hole pairs at a rate $A(\tau)$
- e^--h pair recombine at a rate $B(\tau)\, np$

$$\frac{dn}{dt} = \frac{dp}{dt} = A(\tau) - B(\tau)\, np$$

At equilibrium $\frac{dn}{dt} = \frac{dp}{dt} = 0 \Rightarrow A(\tau) = B(\tau)\, np$

$np = \frac{A(\tau)}{B(\tau)}$: fixed for constant T.

Increase n (by putting impurities in the crystal) has to reduce p.

With impurities we can reduce the total number of carriers $n+p$.

without impurities $n_i = p_i = \sqrt{np}$

$$n = p = 2\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2} (m_e m_h)^{3/4} \exp\left[-\frac{E_g}{2k_B T}\right]$$

$$= 2\left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{3/2} \exp\left[\frac{\mu - E_c}{k_B T}\right]$$

$$= 2\left(\frac{m_h k_B T}{2\pi \hbar^2}\right)^{3/2} \exp\left[\frac{E_v - \mu}{k_B T}\right]$$

$$2\left(\frac{m_h k_B T}{2\pi \hbar^2}\right)^{3/2} \exp\left[\frac{E_v - \mu}{k_B T}\right] = 2\left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{3/2} \exp\left[\frac{\mu - E_c}{k_B T}\right]$$

$$m_h^{3/2} e^{E_v/k_B T} e^{-\mu/k_B T} = m_e^{3/2} e^{\mu/k_B T} e^{-E_c/k_B T}$$

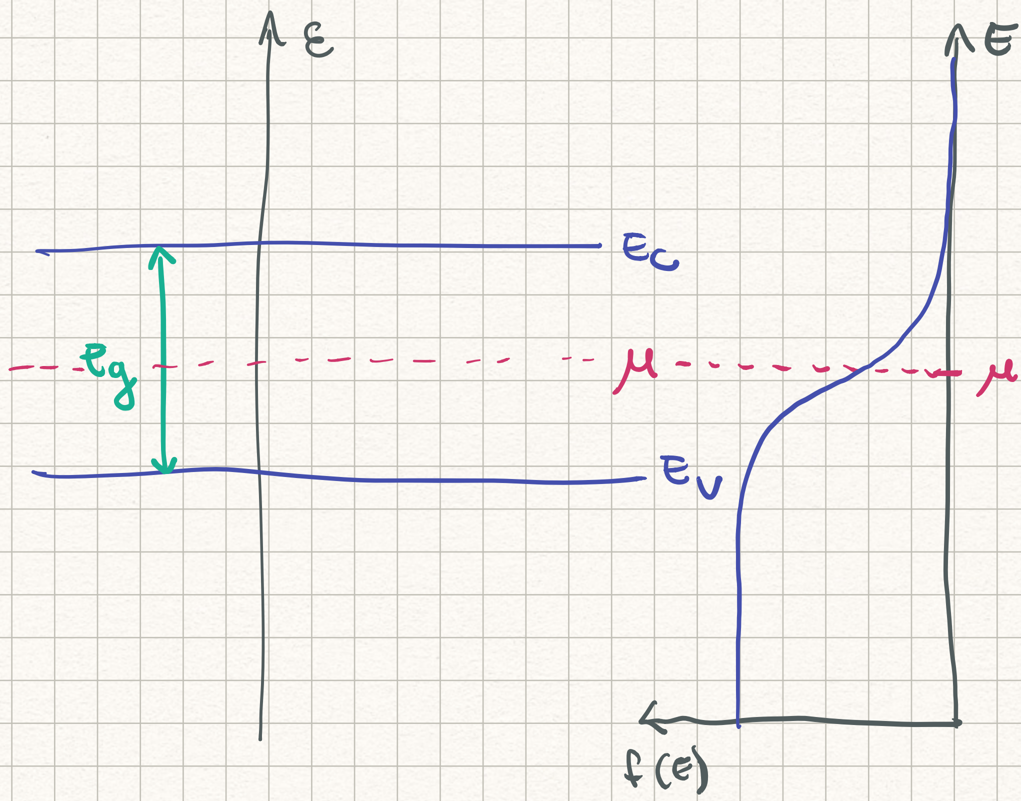
$$\left(\frac{m_h}{m_e}\right)^{3/2} e^{(E_c + E_v)/k_B T} = e^{2\mu/k_B T}$$

$$\ln\left(\left(\frac{m_h}{m_e}\right)^{3/2} e^{(E_c + E_v)/k_B T}\right) = \ln\left(e^{2\mu/k_B T}\right)$$

$$\frac{3}{2}\ln\left(\frac{m_h}{m_e}\right) + \frac{E_c + E_v}{k_B T} = \frac{2\mu}{k_B T}$$

$$\boxed{\mu = \frac{3}{4} k_B T \ln\left(\frac{m_h}{m_e}\right) + \frac{E_c + E_v}{2}}$$

If $m_e = m_h \Rightarrow \mu = \frac{E_c + E_v}{2}$: The Fermi level is in the midde of the forbidden gap.



ε
E
ε_c
ε_g
μ
μ
ε_v
f(ε)