Motion in Magnetic field

In the presence of impurities (finite collision time τ)

$$\vec{F} = m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\vec{v}$$

The term $m\frac{d\vec{v}}{dt} \Rightarrow$ free acceleration

The term $m\vec{v}/\tau \Rightarrow$ effect of collisions $\Rightarrow$ a friction

In the presence of a $\vec{B}$-field

$$\vec{F} = -e(\vec{E} + \vec{v}\times\vec{B}) = m\left(\frac{d}{dt} + \frac{1}{\tau}\right)\vec{v}$$

In the steady-state, $d\vec{v}/dt = 0$, we have

$$\frac{m}{\tau}\vec{v} = -e(\vec{E} + \vec{v}\times\vec{B})$$

$$\vec{v} = -\frac{\tau e}{m}(\vec{E} + \vec{v}\times\vec{B})$$

Particular case $\vec{B} = B\hat{k}$

In this case

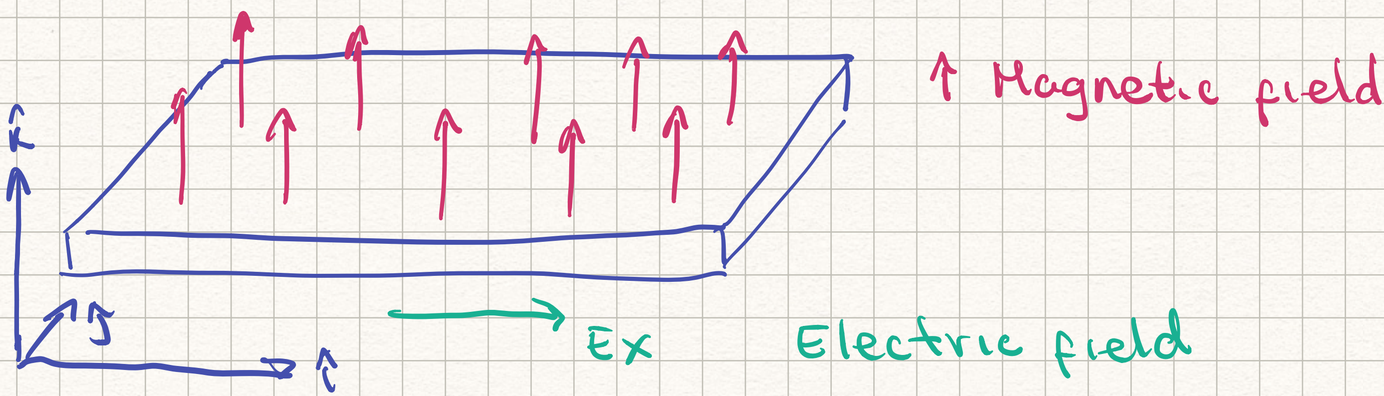
$$\vec{v}\times\vec{B} = (Bv_y, -Bv_x, 0)$$

$$v_x = -\frac{\tau e}{m}E_x - \frac{\tau e}{m}Bv_y$$

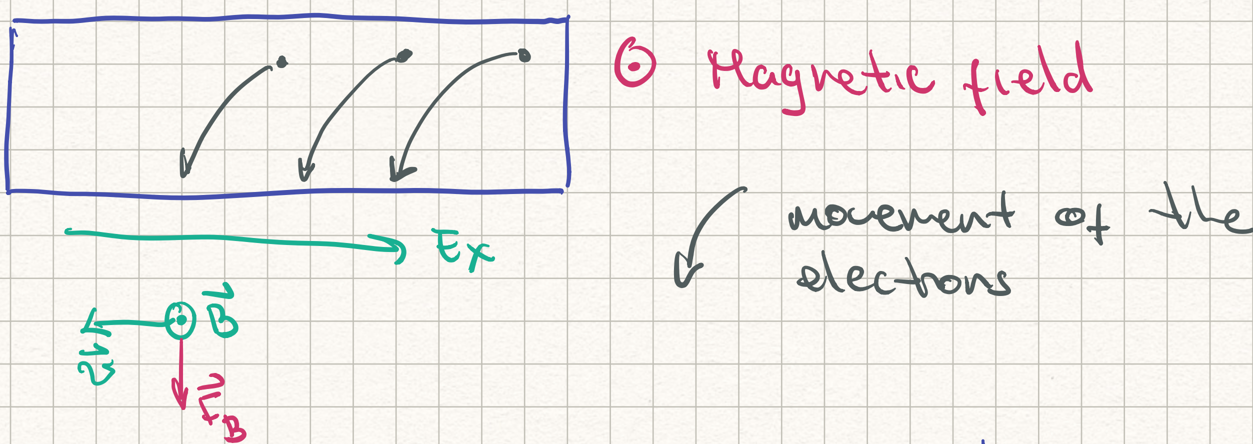
$$v_y = -\frac{\tau e}{m}E_y + \frac{\tau e}{m}Bv_x$$

$$v_z = -\frac{\tau e}{m}E_z$$

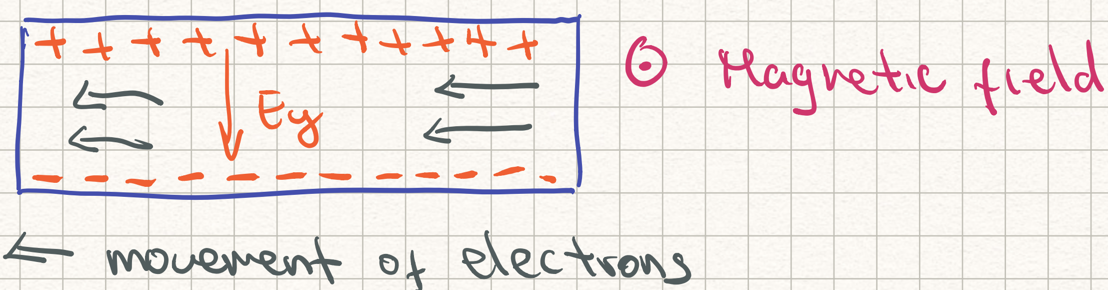
Let's consider a sample like this on



- A magnetic field accross the sample $\vec{B} = B\hat{k}$
- An electric field E_x applied transverse to the $\vec{B}$-field. This induces a current along $\hat{i}$



The electrons moving because of the $\vec{E}$-field are diverte towords the edge of the sample because of the $\vec{B}$-field



In this case $v_y = 0$

$$v_x = -\frac{\tau e}{m} E_x$$

$$0 = -\frac{\tau e}{m} E_y + \frac{\tau e}{m} B v_x$$

$E_y = B v_x$

$$\boxed{E_y = -\frac{\tau e B}{m} E_x}$$

The Hall effect

using $\vec{J} = \sigma \vec{E} = \left(\frac{n e^2 \tau}{m}\right) \vec{E} \Rightarrow E_x = \left(\frac{m}{n e^2 \tau}\right) J_x$

$$E_y = -\frac{B e \tau}{m} E_x = -\frac{B e \tau}{m}\left(\frac{m}{n e^2 \tau}\right) J_x$$

$$= -\frac{B}{ne} J_x \Rightarrow \boxed{\frac{E_y}{B J_x} = -\frac{1}{ne}}$$

Hall coefficient

$R_H = -\frac{1}{ne}$

R_H is larger as the density of carriers is reduced. Thus, measuring R_H is important to deduce the density of carriers.

Energy gaps

Let's consider a 1D crystal with lattice constant a.

Use the PBM to solve Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_n(x) = E_n \psi_n(x)$$

we find $\psi_n(x) = A_n e^{ikx}$ with

$$k = 0 ; \pm \frac{2\pi}{a} ; \pm \frac{4\pi}{a} , \dots$$

with $E_n = \frac{\hbar^2 k^2}{2m}$.

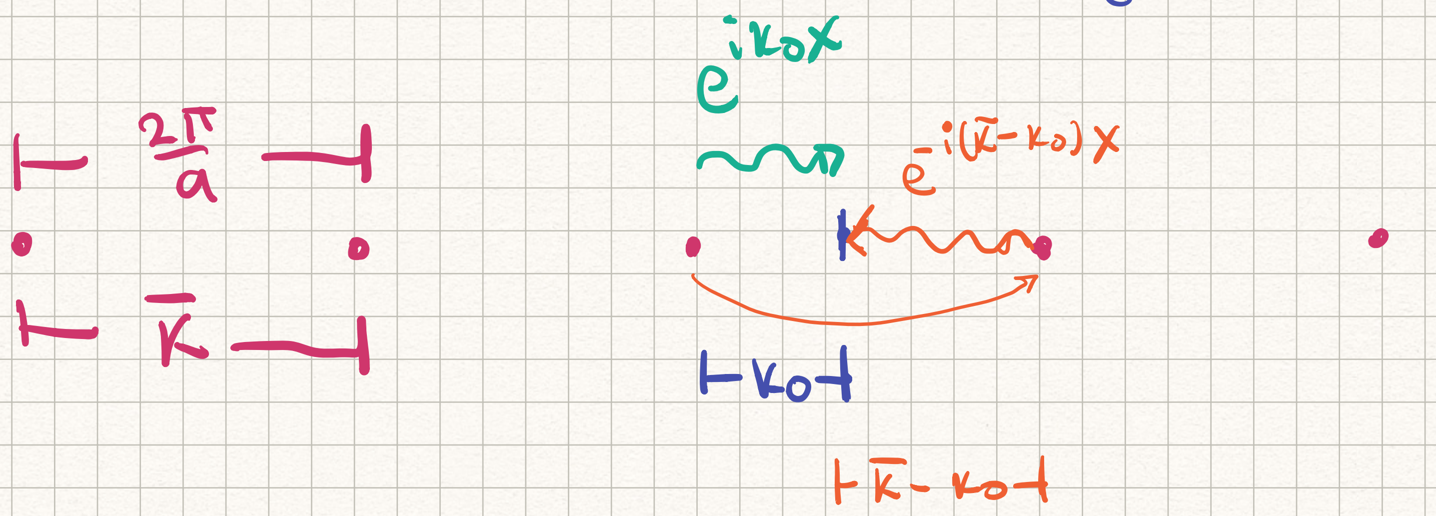
$\psi_n(x) = A_n e^{ikx}$ is a travelling wave.

As the lattice has a periodicity a, the reciprocal lattice has a periodicity $2\pi/a$.

This means that for every x, we can express the WF as

$$\psi_n(x) = A_n e^{ik_0 x} \quad \text{or} \quad \psi_n(x) = A_n e^{-i(\bar{k}-k_0)x}$$

for some wavevector k_0 and using $\bar{k} = 2\pi/a$.



The sum of the two WF is also a solution and it's a valid WF

$$\psi_\pm(x) = A_n e^{ik_0 x} \pm A_n e^{-i(\bar{k}-k_0)x}$$

In general the WF is also a travelling wave. However, at $k_0 = \bar{k}/2$

$$\psi_\pm(x) = A_n \left(e^{i\bar{k}x/2} \pm e^{-i\bar{k}x/2}\right)$$

These are standing waves $= \begin{cases} 2A_n \cos(\bar{k}x/2) & \text{if } (+) \\ 2A_n i \sin(\bar{k}x/2) & \text{if } (-) \end{cases}$

This means that at $k_0 = \bar{k}/2$, at any point in space (x), a wave travelling to the right is reflected to the left and viceversa.

What is special about $k_0 = \bar{k}/2$?

Let's remember the diffraction condition

$$(\vec{k} + \vec{G})^2 = k^2$$

In 1D, $\vec{G} = \pm \frac{2\pi}{a} n \hat{\imath}$ $\qquad \vec{k} = k\hat{\imath}$

$$(\vec{k}+\vec{G})\cdot(\vec{k}+\vec{G}) = k^2 + G^2 + 2\vec{k}\cdot\vec{G}$$
$$= k^2 + \left(\frac{2\pi}{a}\right)^2 n^2 + \frac{2\pi n k}{a} = k^2$$

$$k = \pm \frac{1}{2}\left(\frac{2\pi n}{a}\right) = \pm \frac{n\pi}{a} = \pm \frac{n\bar{k}}{2} \qquad \bar{k} = \frac{2\pi}{a}$$

At $k_0 = \bar{k}/2$ $(n=1)$ we are at the edge of the 1st Brillouin zone

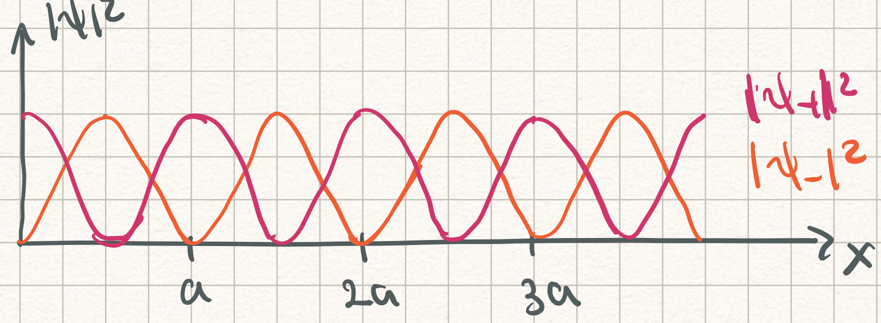
At the borders of the Brillouin zone the waves do not propagate, but are rather reflected; thus generating standing waves

$\psi_+(x) = 2A_n \cos(\bar{k}x/2) \Rightarrow$ $\quad \bar{k} = \frac{2\pi}{a} \quad$ $\psi_+ = 2\cos(\pi x/a) A_n$

$\psi_-(x)\ 2A_n i \sin(\bar{k}x/2) \Rightarrow$ $\quad$ $\psi_- = 2iA_n \sin(\pi x/a)$

The density of probability to find an e^- at position x is given $|\psi(x)|^2$

$$|\psi_+|^2 \propto \cos^2(\pi x/a)$$
$$|\psi_-|^2 \propto \sin^2(\pi x/a)$$



$|\psi_+|^2$ piles the e^- at the positive ions, while $|\psi_-|^2$ piles them <u>away</u> from the ions