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Preparation of a three-photon state in a nonlinear cavity–quantum dot system

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Abstract

We study theoretically the properties of a three-photon state prepared inside a semiconductor cavity, due to the interaction between a quantum dot and an electromagnetic field, and two consecutive spontaneous parametric downconversion (SPDC) processes. Thus, we consider a scheme involving three modes of the electromagnetic field, whose frequencies are given by the SPDC processes: $\omega_0 \rightarrow \omega_1 + \omega_2$ and $\omega_2 \rightarrow \omega_1 + \omega_1$. Furthermore, we study the low excitation regime in which a three-photon state is accessible within the system's dynamics.

Keywords: three-photon state, nonlinear cavity, quantum dot, spontaneous parametric downconversion process, photonic crystal

(Some figures may appear in colour only in the online journal)

1. Introduction

During the last few years, several research groups have been studying the light–matter interaction in quantum dots (QDs) embedded in semiconductor microcavities, both experimentally [1–6] and theoretically [7–15]. Such investigations led to a new phenomenology which in turn has led to technological applications [16–21].

On the other hand, the generation of photon n -plets has been an interesting research branch, because it could allow researchers to prepare quantum states inside cavities which would be useful in quantum communication [22]. In particular, three-photon states can be obtained experimentally using spontaneous parametric downconversion (SPDC) [23–28], and third-order optical nonlinearities in assembled [29–31] systems. In this sense, although several groups have managed to prepare and control specific quantum states [32, 33], the preparation of arbitrary quantum states of light is still an experimental challenge.

Bearing in mind the cavity quantum electrodynamics (cQED) description of the light–matter interactions and the preparation of quantum states of light via SPDC processes, we consider that it is possible to set up an experimental design in which the initial state inside a cavity can be prepared, and finely controlled. Even though the cavity does not have to be microscopic, the experimental design is scalable from those of semiconductor microcavities. This means that the problem

and the obtained results are not restricted to the optical region of the electromagnetic spectrum. Therefore, in order to prepare the quantum state inside a cavity, a mesoscopic nonlinear crystal can be included in the experimental design. On the other hand, it has recently been demonstrated that photonic crystal (PhC) cavities are capable of enhancing the harmonic generation produced by either a $\chi^{(2)}$ or a $\chi^{(3)}$ nonlinearity, which would finally yield an SPDC process [34–37]. Furthermore, it has been shown that the adequate geometry of the PhC [34], pump power [35] and whether the cavity is singly or doubly resonant [36] may lead to a 100% conversion.

In this sense, we consider a semiconductor cavity in which there are a QD and two nonlinear crystals. The former is coupled to a ω_0 electromagnetic mode, so the cavity is filled with ω_0 photons. Afterwards, these photons go through the nonlinear crystals and two SPDC processes take place: $\omega_0 \rightarrow \omega_1 + \omega_2$ and $\omega_2 \rightarrow \omega_1 + \omega_1$. Taking into account an exciton pumping and ω_0 photon leakage from the cavity, which are both incoherent processes, a three-photon state is accessible in the ω_1 electromagnetic mode.

The nonlinear cavity–QD system can be constructed using a GaAs substrate, over which several layers of $\text{Al}_x\text{Ga}_{1-x}\text{As}$, and a layer of $\text{Al}_y\text{In}_{1-y}\text{As}$ in which the QDs are localized, are grown using the molecular beam epitaxy (MBE) method. In this particular construction, the optical properties are nonlinear [37, 38], and could therefore be used as a basis

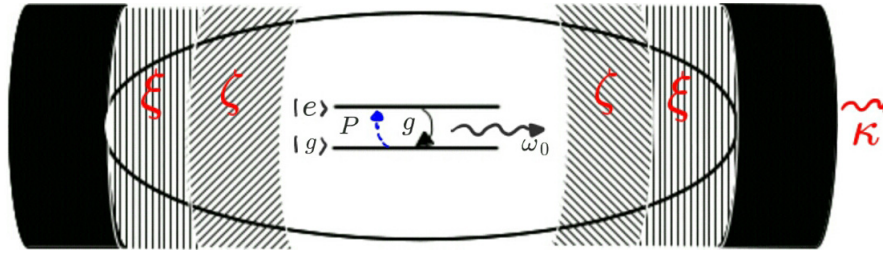


Figure 1. Diagram of the physical system. The nonlinear crystals (ζ and ξ) yield the SPDC processes.

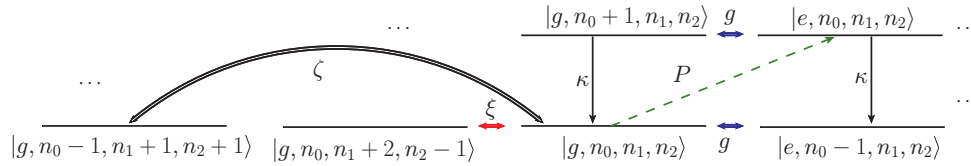


Figure 2. Ladder of energy levels for the QD–cavity system accessible from the $|g, n_0, n_1, n_2\rangle$ state by just one process. The energy levels are depicted by straight continuous lines corresponding to quantum states. The blue double lines correspond to the interaction between the QD and the ω_0 mode. The black (red) double lines connect the accessible states via the SPDC process associated with ζ (ξ). The continuous black lines describe the ω_0 leakage process, whereas the green piecewise line corresponds to the incoherent exciton pumping.

for the experimental design of our system. Other such systems consist of PhC made of periodically poled lithium niobate (PPLN) [28, 39] or periodically poled potassium titanyl phosphate (KTP) [40], which give rise to an enhancement of the harmonic generation.

A possible drawback of this kind of experimental design lies in the fact that the inclusion of QD into nonlinear cavities yield physical phenomena such as the Kerr effect (due to the presence of other QDs in the cavity), the harmonic generation or the Purcell effect (due to the spontaneous emission from a dipole source). Nevertheless, it has been shown that PhC cavities can lead to an enhancement of the nonlinear phenomena and suppress the spontaneous emission via a photonic band gap, which increases the $\chi^{(3)}$ nonlinearity [35]. Furthermore, the SPDC processes can achieve 100% efficiency by using a doubly resonant cavity [36], but producing such cavities is a challenge because they require confinement at two very different frequencies [35].

The rest of the paper is organized as follows. In section 2, we present the model. In section 3, we present our results and discuss their consequences. Finally, in section 4 we provide an overview of the results and conclude.

2. Model

Our model considers the interaction between a QD and an electromagnetic mode inside a semiconductor cavity, followed by two consecutive SPDC processes. The latter lead to a total of three electromagnetic modes. The QD's elementary excitations—*excitons*—are the result of an electron being promoted to the conduction from the valence band. In this paper, we model the interaction between the excitons and an electromagnetic mode with the usual Jaynes–Cummings model [41, 42], whose Hamiltonian is the following (\hbar is taken as 1 throughout the paper):

$$H_{\text{JC}} = \omega_0 a_0^\dagger a_0 + \omega_{\text{qd}} \sigma^\dagger \sigma + g \left(a_0^\dagger \sigma + a_0 \sigma^\dagger \right), \quad (1)$$

where $a_0(a_0^\dagger)$ is the ω_0 electromagnetic mode annihilation (creation) operator and $\sigma(\sigma^\dagger)$ is the exciton annihilation (creation) operator. The ω_0 electromagnetic mode and the QD's exciton are coupled with an interaction strength g and their frequencies are close enough to resonance to allow for the rotating wave approximation, i.e. $\Delta = \omega_0 - \omega_{\text{qd}} \ll \omega_0, \omega_{\text{qd}}$ [43].

The two subsequent SPDC processes generate two more electromagnetic modes with frequencies ω_1 and ω_2 . In the first process, one ω_0 photon generates one ω_1 and one ω_2 photon ($\omega_0 \rightarrow \omega_1 + \omega_2$), whereas in the second process one ω_2 generates two ω_1 photons ($\omega_2 \rightarrow \omega_1 + \omega_1$). Both processes may be described in an effective way with the following Hamiltonian [30]:

$$H_{\text{SPDC}} = \zeta \left(a_0 a_1^\dagger a_2^\dagger + a_0^\dagger a_1 a_2 \right) + \xi \left(a_1^{\dagger 2} a_2 + a_1^2 a_2^\dagger \right), \quad (2)$$

where ζ and ξ are the rates at which the processes occur and $a_i(a_i^\dagger)$ are the ω_i mode annihilation (creation) operators. The physical system is depicted in figure 1 and is described by the Jaynes–Cummings plus SPDC Hamiltonians,

$$H = H_{\text{JC}} + H_{\text{SPDC}}. \quad (3)$$

The dynamical behavior and the incoherent pumping and loss of the dot–cavity system are included in the master equation, which in the Lindblad notation is written as

$$\dot{\rho} = i[\rho, H] + \frac{P}{2} (2\sigma^\dagger \rho \sigma - \{\sigma \sigma^\dagger, \rho\}) + \frac{\kappa}{2} (2a_0 \rho a_0^\dagger - \{a_0^\dagger a_0, \rho\}), \quad (4)$$

where H is the Hamiltonian given in equation (3), κ is the rate at which ω_0 photons escape from the cavity and P is the rate at which the excitation is pumped to the cavity and is linked to the rate at which electron–hole pairs relax into the dot.

Furthermore, the system's energy levels and its connection via the master equation given in equation (4) are shown schematically in figure 2. Each energy level is

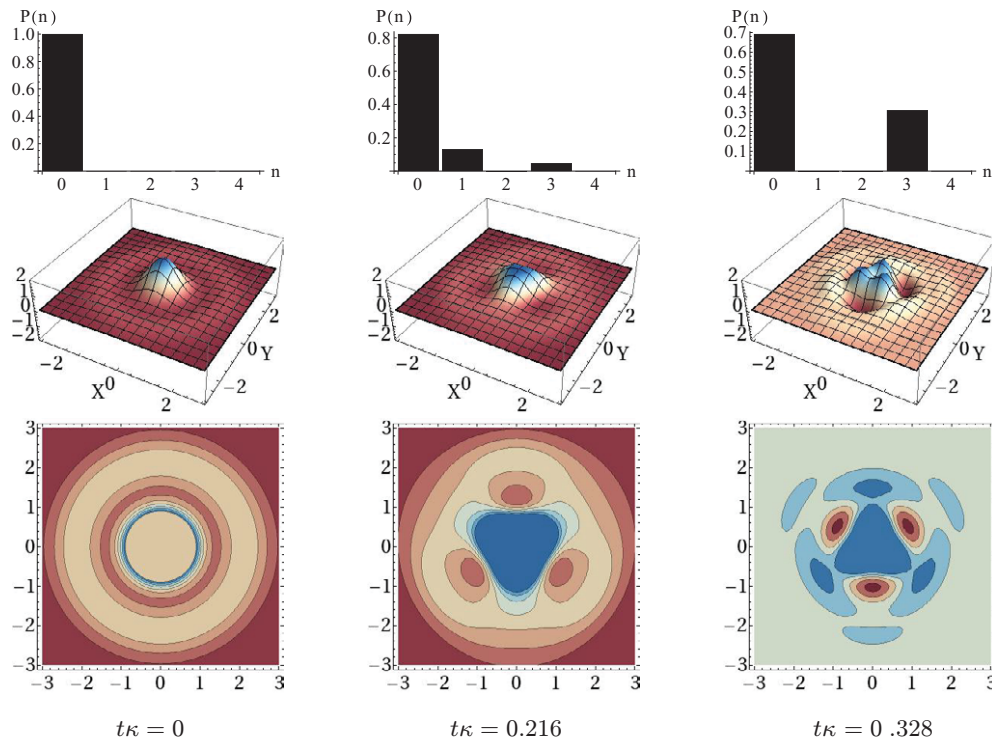


Figure 3. Photon number distribution (first row), Wigner functions (second row) and its contour plots (third row) for the ω_1 mode for three different time intervals. The parameters used are $g/\kappa = 50$, $\zeta/\kappa = 30$, $\xi/\kappa = 10$, $\kappa/P = 1000$, $\omega_0 = \omega_{\text{qd}} = 500$ meV and $g = 5$ meV.

associated with a quantum state written as $|a, i, j, k\rangle$, where a is the QD state (either ground or excited) and i, j and k are the photon numbers in the ω_0, ω_1 and ω_2 modes of the electromagnetic field, respectively. The presented scheme shows the energy levels accessible for the $|g, n_0, n_1, n_2\rangle$ state by just one process.

3. Results

We solved the master equation given in (4) numerically using the following parameter values: the dipole-like interaction constant between the QD and the ω_0 mode is set as $g = 5$ meV; the excitation energy of the QD is set as $\omega_{\text{qd}} = 500$ meV which in turn is tuned perfectly with the ω_0 mode, i.e. $\omega_{\text{qd}} = \omega_0$. These values are usual for λ cavities operating in the infrared region of the electromagnetic spectrum. Furthermore, restricting our results to the low-excitation regime, we set the incoherent pumping rate P as $0.1 \mu\text{eV}$, whereas the cavity loss for the fundamental mode ω_0 is taken as $\kappa = 0.1$ meV. On the other hand, the SPDC rates are taken as $\xi = 1$ meV and $\zeta = 3$ meV, following the recommendations presented in [30].

We consider the QD in its excited state and the electromagnetic field to be in a vacuum state in all of its modes, as the system's initial condition. With this setup, we observe that the state of the ω_1 electromagnetic mode reaches a so-called three-photon state, which is a superposition of $3n$ -photon Fock states, within the system's dynamics. These results are shown in figure 3.

In this way, we have obtained results similar to those presented in [30], considering explicitly the

interaction between a QD in a semiconductor cavity and an electromagnetic field. These results are very interesting, since we have shown that a three-photon state can be prepared inside a semiconductor cavity made of PhC capable of enhancing the harmonic generation produced by either a $\chi^{(2)}$ or $\chi^{(3)}$ nonlinearity.

4. Summary and conclusions

In this paper, we have studied theoretically the preparation of a three-photon state inside a semiconductor cavity made of PhC capable of enhancing the harmonic generation produced by either a $\chi^{(2)}$ or $\chi^{(3)}$ nonlinearity. The three-photon state is the result of the interaction between a QD embedded in the cavity and a ω_0 mode of the electromagnetic field, and two SPDC processes yielding two more modes of the electromagnetic field: $\omega_0 \rightarrow \omega_1 + \omega_2$ and $\omega_2 \rightarrow \omega_1 + \omega_1$. To study the system's dynamics we have solved a Lindblad master equation numerically considering both an incoherent excitation pump rate and ω_0 -photon leakage from the cavity. In this way, we observed that the three-photon state is accessible within the dynamics in the ω_1 mode in a low-excitation regime.

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Appendix. Dynamics of the density operator's matrix elements

On the basis $\{|g, n_0, n_1, n_2\rangle; |e, n_0, n_1, n_2\rangle\}$ of product states between the QD and the ω_0 , ω_1 and ω_2 electromagnetic modes, the matrix elements of the density operator are

$$\rho_{a,i,j,k;b,l,m,n} = \langle a, i, j, k | \rho | b, l, m, n \rangle, \quad (\text{A.1})$$

where a and b are either g or e .

In this notation, the density operator's matrix elements satisfy the following differential equations:

$$\begin{aligned} \partial_t \rho_{g,i,j,k;g,l,m,n} = & \left[i \omega_0 \left(l - i + \frac{m-j}{3} + 2 \frac{n-k}{3} \right) - \kappa \frac{l+i}{2} - P \right] \\ & \times \rho_{g,i,j,k;g,l,m,n} \\ & + i g \left(\sqrt{l} \rho_{g,i,j,k;e,l-1,m,n} - \sqrt{i} \rho_{e,i-1,j,k;g,l,m,n} \right) \\ & + \kappa \sqrt{(i+1)(l+1)} \rho_{g,i+1,j,k;g,l+1,m,n} \\ & + i \zeta \left(\sqrt{l(m+1)(n+1)} \rho_{g,i,j,k;g,l-1,m+1,n+1} \right. \\ & \left. + \sqrt{(l-1)mn} \rho_{g,i,j,k;g,l+1,m-1,n-1} \right) \\ & - i \zeta \left(\sqrt{(i+1)jk} \rho_{g,i+1,j-1,k-1;g,l,m,n} \right. \\ & \left. + \sqrt{i(j+1)(k+1)} \rho_{g,i-1,j+1,k+1;g,l,m,n} \right) \\ & + i \xi \left(\sqrt{m(m-1)(n+1)} \rho_{g,i,j,k;g,l,m-2,n+1} \right. \\ & \left. + \sqrt{(m+1)(m+2)n} \rho_{g,i,j,k;g,l,m+2,n-1} \right) \\ & - i \xi \left(\sqrt{(j+1)(j+2)k} \rho_{g,i,j+2,k-1;g,l,m,n} \right. \\ & \left. + \sqrt{j(j-1)(k+1)} \rho_{g,i,j-2,k+1;g,l,m,n} \right), \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} \partial_t \rho_{e,i,j,k;e,l,m,n} = & \left[i \omega_0 \left(l - i + \frac{m-j}{3} + 2 \frac{n-k}{3} \right) - \kappa \frac{l+i}{2} \right] \\ & \times \rho_{e,i,j,k;e,l,m,n} + P \rho_{g,i,j,k;g,l,m,n} \\ & + i g \left(\sqrt{l+1} \rho_{e,i,j,k;g,l+1,m,n} - \sqrt{i+1} \rho_{g,i+1,j,k;e,l,m,n} \right) \\ & + \kappa \sqrt{(i+1)(l+1)} \rho_{e,i+1,j,k;e,l+1,m,n} \\ & + i \zeta \left(\sqrt{l(m+1)(n+1)} \rho_{e,i,j,k;e,l-1,m+1,n+1} \right. \\ & \left. + \sqrt{(l-1)mn} \rho_{e,i,j,k;e,l+1,m-1,n-1} \right) \\ & - i \zeta \left(\sqrt{(i+1)jk} \rho_{e,i+1,j-1,k-1;e,l,m,n} \right. \\ & \left. + \sqrt{i(j+1)(k+1)} \rho_{e,i-1,j+1,k+1;e,l,m,n} \right) \\ & + i \xi \left(\sqrt{m(m-1)(n+1)} \rho_{e,i,j,k;e,l,m-2,n+1} \right. \\ & \left. + \sqrt{(m+1)(m+2)n} \rho_{e,i,j,k;e,l,m+2,n-1} \right) \end{aligned}$$

$$\begin{aligned} & - i \xi \left(\sqrt{(j+1)(j+2)k} \rho_{e,i,j+2,k-1;e,l,m,n} \right. \\ & \left. + \sqrt{j(j-1)(k+1)} \rho_{e,i,j-2,k+1;e,l,m,n} \right), \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \partial_t \rho_{g,i,j,k;e,l,m,n} = & \left[i \omega_0 \left(l - i + \frac{m-j}{3} + 2 \frac{n-k}{3} \right) \right. \\ & \left. + i \omega_{\text{qd}} - \kappa \frac{l+i}{2} - \frac{P}{2} \right] \rho_{g,i,j,k;e,l,m,n} \\ & + i g \left(\sqrt{l+1} \rho_{g,i,j,k;g,l+1,m,n} - \sqrt{i} \rho_{e,i-1,j,k;e,l,m,n} \right) \\ & + \kappa \sqrt{(i+1)(l+1)} \rho_{g,i+1,j,k;e,l+1,m,n} \\ & + i \zeta \left(\sqrt{l(m+1)(n+1)} \rho_{g,i,j,k;e,l-1,m+1,n+1} \right. \\ & \left. + \sqrt{(l-1)mn} \rho_{g,i,j,k;e,l+1,m-1,n-1} \right) \\ & - i \zeta \left(\sqrt{(i+1)jk} \rho_{g,i+1,j-1,k-1;e,l,m,n} \right. \\ & \left. + \sqrt{i(j+1)(k+1)} \rho_{g,i-1,j+1,k+1;e,l,m,n} \right) \\ & + i \xi \left(\sqrt{m(m-1)(n+1)} \rho_{g,i,j,k;e,l,m-2,n+1} \right. \\ & \left. + \sqrt{(m+1)(m+2)n} \rho_{g,i,j,k;e,l,m+2,n-1} \right) \\ & - i \xi \left(\sqrt{(j+1)(j+2)k} \rho_{g,i,j+2,k-1;e,l,m,n} \right. \\ & \left. + \sqrt{j(j-1)(k+1)} \rho_{g,i,j-2,k+1;e,l,m,n} \right), \end{aligned} \quad (\text{A.4})$$

plus the Hermitian conjugate of (A.4).

Once the system of linear equations is solved, we obtain the density operator of the cavity-QD system as a function of time: $\rho(t)$. This operator has four quantum numbers associated, one to the QD and one to each of the modes of the electromagnetic field, and its matrix elements are thus given by

$$\rho_{a,i,j,k;b,l,m,n}(t) = \langle a, i, j, k | \rho(t) | b, l, m, n \rangle. \quad (\text{A.5})$$

Nevertheless, in this particular case we are only interested in the degree of freedom associated with the ω_1 mode, so it is convenient to consider the reduced (to the ω_1 subsystem) density operator instead of the complete operator. The reduced operator is denoted as $\rho^{(3)}(t)$, and is obtained from the complete operator by performing partial trace over all the remaining degrees of freedom:

$$\rho_{i,j}^{(3)}(t) = \langle i | \rho^{(3)}(t) | j \rangle = \sum_{a,n,m} \langle a, n, i, m | \rho(t) | a, n, j, m \rangle. \quad (\text{A.6})$$

Finally, once we have obtained the reduced density operator for the ω_1 mode, we compute its Wigner function as in e.g. [44].

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